

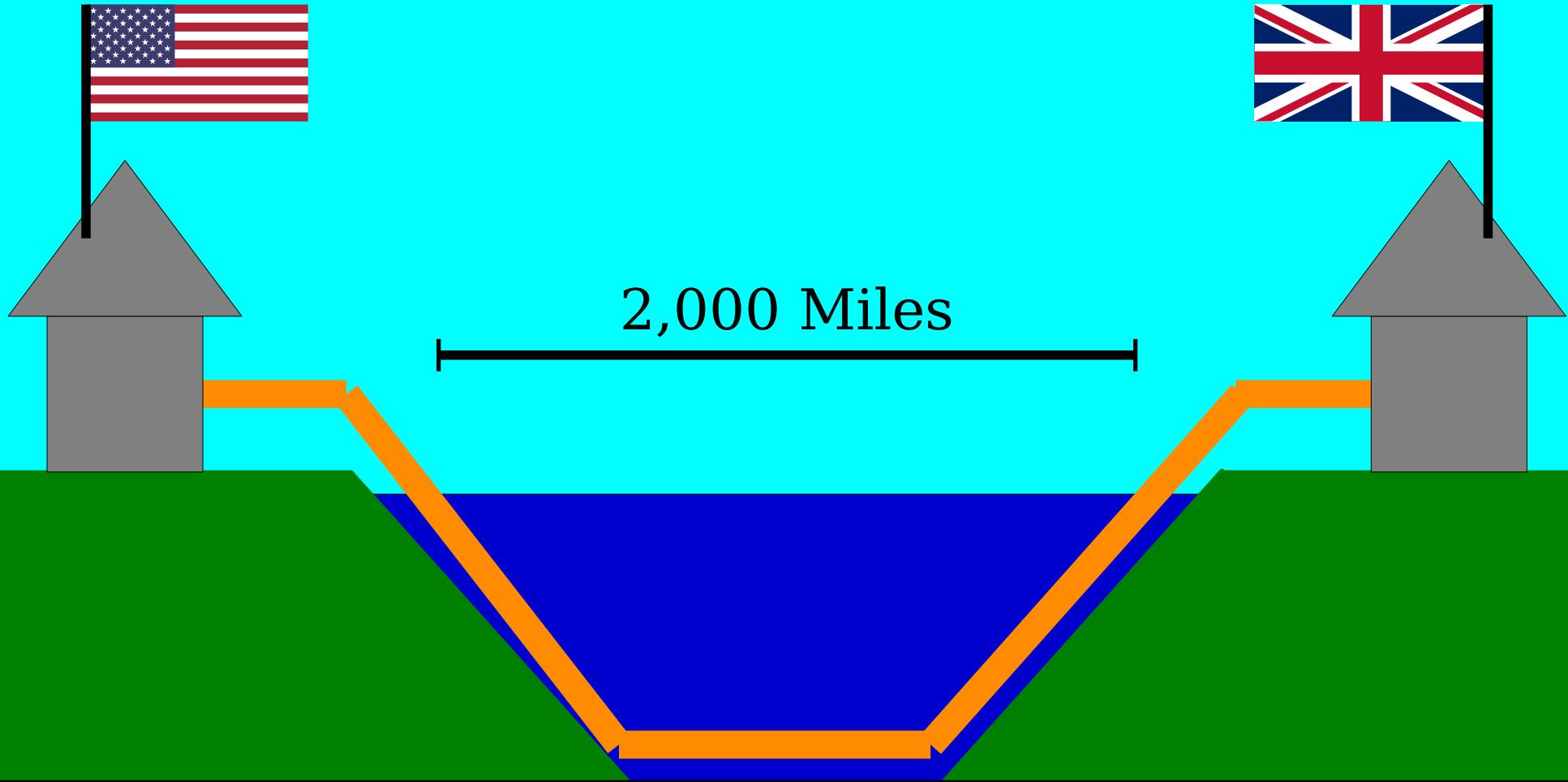
# Beyond Data Structures

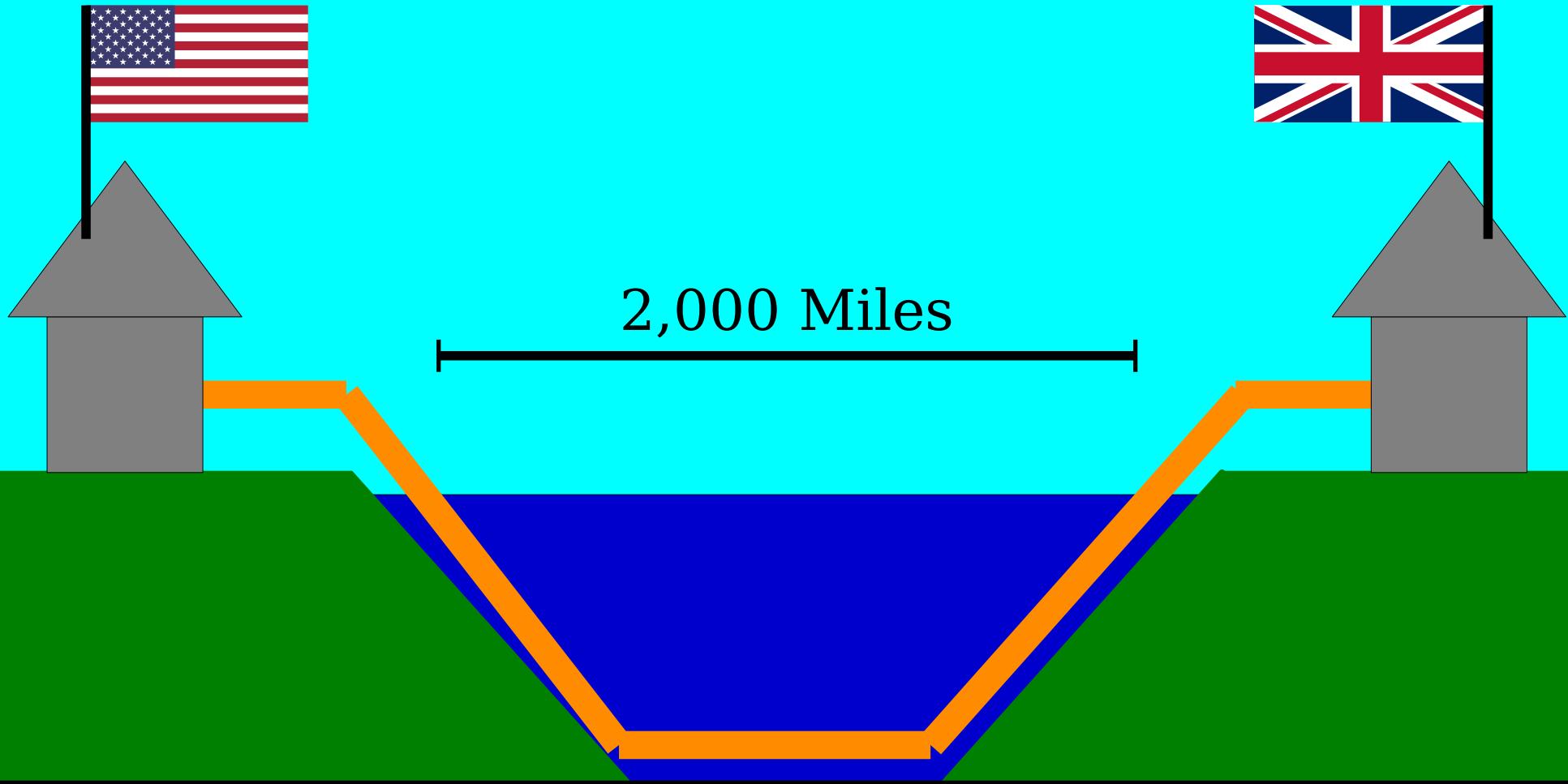
# Outline for Today

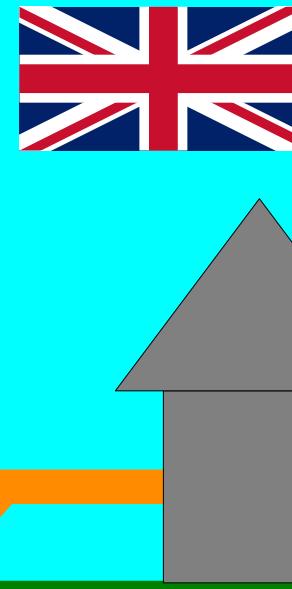
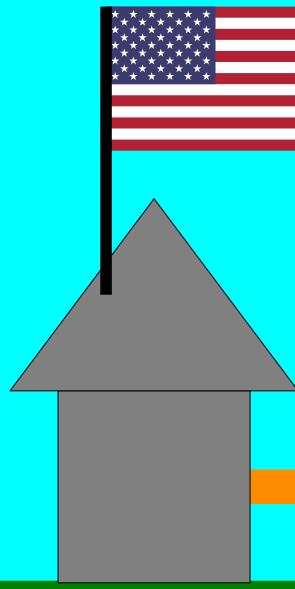
- ***Bits and Bytes***
  - Representing things with 0s and 1s.
- ***Data Compression***
  - Reducing transmission requirements.
- ***Prefix-Free Codes***
  - A clever space-saving trick.
- ***Huffman Coding***
  - Finding good prefix-free codes.

# Bits and Bytes

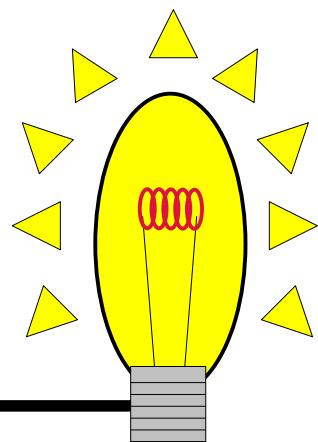
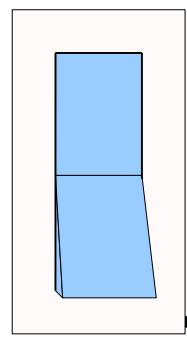
# 19<sup>th</sup> Century Data Transmission

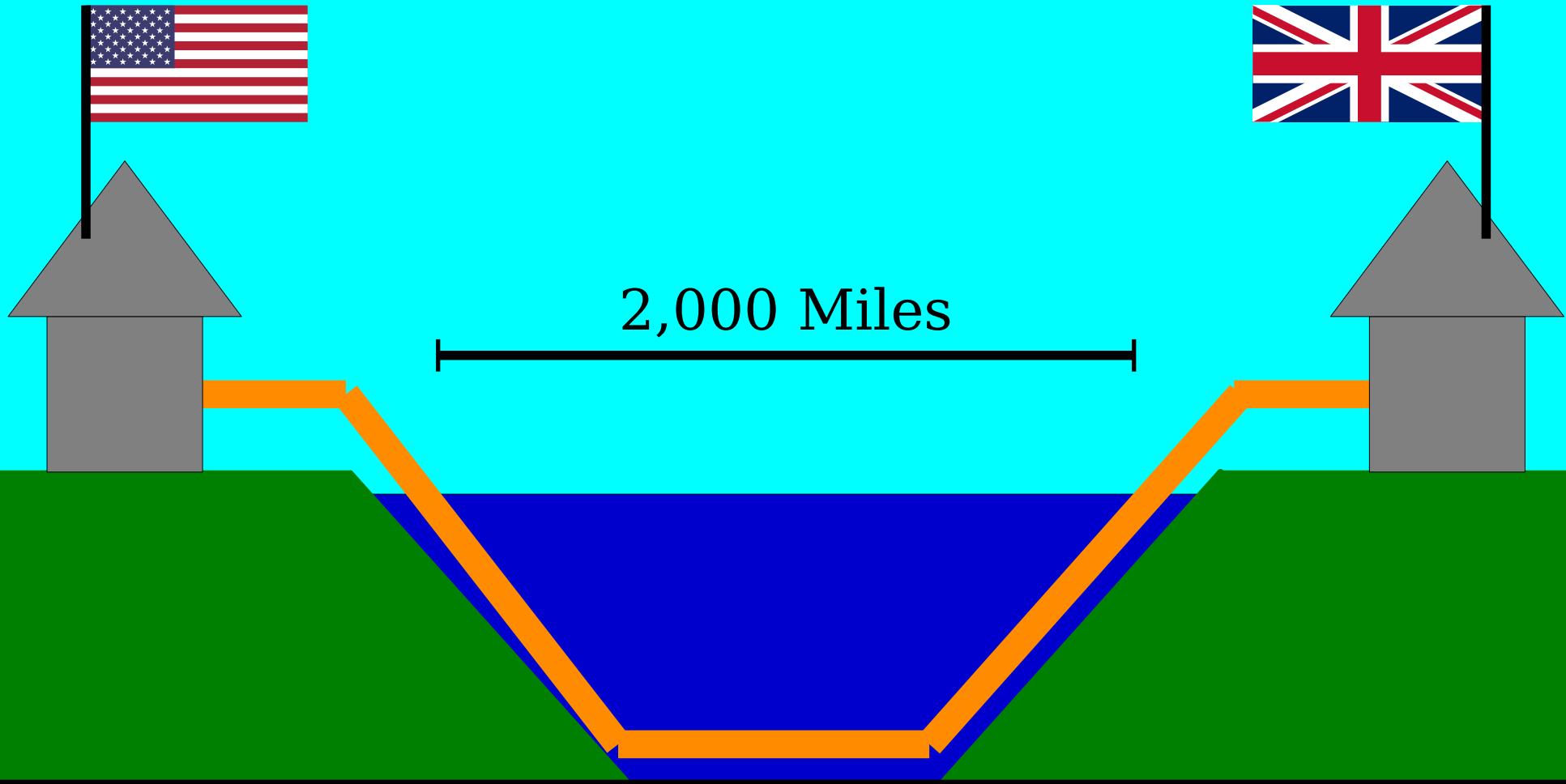






2,000 Miles



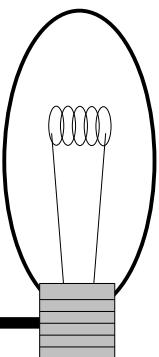
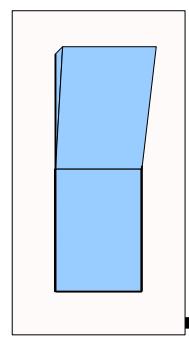


A	• -
B	- • • •
C	- • - •
D	- • •
E	•
F	• • - •
G	- - •

H	• • • •
I	• •
J	• - - -
K	- • -
L	• - • •
M	- -
N	- •

O	- - -
P	• - - •
Q	- - • -
R	• - •
S	• • •
T	-
U	• • -

V	• • • -
W	• - -
X	- • • -
Y	- • - -
Z	- - • •

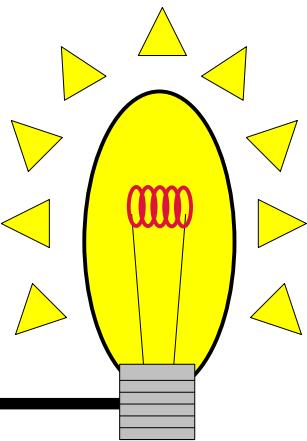
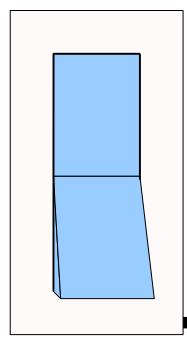


A	• -
B	- • • •
C	- • - •
D	- • •
E	•
F	• • - •
G	- - •

H	• • • •
I	• •
J	• - - -
K	- • -
L	• - • •
M	- -
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O	- - -
P	• - - •
Q	- - • -
R	• - •
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W	• - -
X	- • • -
Y	- • - -
Z	- - • •

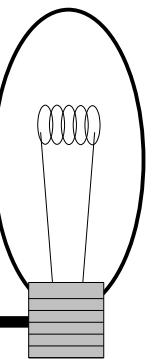
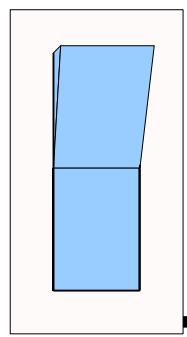


A	• -
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C	- • - •
D	- • •
E	•
F	• • - •
G	- - •

H	• • • •
I	• •
J	• - - -
K	- • -
L	• - • •
M	- -
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R	• - •
S	• • •
T	-
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X	- • • -
Y	- • - -
Z	- - • •



A	• -
B	- • • •
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Y	- • - -
Z	- - • •

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A	• —
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M	— —
N	— •

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P	• — — •
Q	— — • —
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S	• • •
T	—
U	• • —

V	• • • —
W	• — —
X	— • • —
Y	— • — —
Z	— — • •

What is the title of this slide?

Answer at

<https://pollev.com/cs106bwin23>

*D*  
— • •

*I*  
• •

*K*  
— • —

*D*  
— • •

*I*  
• •

*K*  
— • —

A	• —
B	— • • •
C	— • — •
D	— • •
E	•
F	• • — •
G	— — •

H	• • • •
I	• •
J	• — — —
K	— • —
L	• — • •
M	— —
N	— •

O	— — —
P	• — — •
Q	— — • —
R	• — •
S	• • •
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U	• • —

V	• • • —
W	• — —
X	— • • —
Y	— • — —
Z	— — • •

An Amazing Read on Telegraphy:  
**"A Web Around the World"**

# 20<sup>th</sup> Century Data Transmission

# It's All Bits and Bytes

- Digital data is stored as sequences of 0s and 1s.
  - They're usually encoded by magnetic orientation on small (10nm!) metal particles or by trapping electrons in nanoscale gates.
- A single 0 or 1 is called a **bit**.
- A group of eight bits is called a **byte**.

00000000, 00000001, 00000010,  
00000011, 00000100, 00000101, ...
- There are  $2^8 = 256$  different bytes.
  - **Great practice:** Write a function to list all of them!

# Representing Text

- We think of strings as being made of characters representing letters, numbers, emojis, etc.
- Internally to the computer, everything is just a series of bits.
- To bridge the gap, we need to agree on some way of representing characters as sequences of bits.
- ***Idea:*** Assign each character a sequence of bits called a ***code***.

# ASCII

- Early (American) computers needed some standard way to send output to their (physical!) printers.
- Since there were fewer than 256 different characters to print (1960's America!), each character was assigned a one-byte value.
- This initial code was called **ASCII**. It still lives on in a modified form as **UTF-8**, which you saw on Assignment 2.
- For example, the letter A is represented by the byte 01000001 (65). You can still see this in C++:

```
cout << int('A') << endl; // Prints 65
```

01001000010001010100000101000100

- Here's a small segment from the ASCII encodings for characters.
- What is the title of this slide?

Answer at  
<https://pollev.com/cs106bwin23>

<i>character</i>	<i>code</i>
A	01000001
B	01000010
C	01000011
D	01000100
E	01000101
F	01000110
G	01000111
H	01001000

01001000010001010100000101000100

- Here's a small segment from the ASCII encodings for characters.
- What is the title of this slide?

	<i>character</i>	<i>code</i>
A		01000001
B		01000010
C		01000011
D		01000100
E		01000101
F		01000110
G		01000111
H		01001000

**H**

01000101010000101000100

- Here's a small segment from the ASCII encodings for characters.
- What is the title of this slide?

	<i>character</i>	<i>code</i>
A		01000001
B		01000010
C		01000011
D		01000100
E		01000101
F		01000110
G		01000111
H		01001000

**H**

010001010100000101000100

- Here's a small segment from the ASCII encodings for characters.
- What is the title of this slide?

	<i>character</i>	<i>code</i>
A		01000001
B		01000010
C		01000011
D		01000100
E		01000101
F		01000110
G		01000111
H		01001000

**H****E**

0100000101000100

- Here's a small segment from the ASCII encodings for characters.
- What is the title of this slide?

<i>character</i>	<i>code</i>
A	01000001
B	01000010
C	01000011
D	01000100
E	01000101
F	01000110
G	01000111
H	01001000

**H****E****0100000101000100**

- Here's a small segment from the ASCII encodings for characters.
- What is the title of this slide?

<i>character</i>	<i>code</i>
A	01000001
B	01000010
C	01000011
D	01000100
E	01000101
F	01000110
G	01000111
H	01001000

**H**

**E**

**A**

01000100

- Here's a small segment from the ASCII encodings for characters.
- What is the title of this slide?

<i>character</i>	<i>code</i>
A	01000001
B	01000010
C	01000011
D	01000100
E	01000101
F	01000110
G	01000111
H	01001000

**H****E****A****01000100**

- Here's a small segment from the ASCII encodings for characters.
- What is the title of this slide?

<i>character</i>	<i>code</i>
A	01000001
B	01000010
C	01000011
D	01000100
E	01000101
F	01000110
G	01000111
H	01001000

**H**

**E**

**A**

**D**

- Here's a small segment from the ASCII encodings for characters.
- What is the title of this slide?

<i>character</i>	<i>code</i>
A	01000001
B	01000010
C	01000011
D	01000100
E	01000101
F	01000110
G	01000111
H	01001000

# An Observation

- In ASCII, every character has exactly the same number of bits in it.
- Any message with  $n$  characters will use up exactly  $8n$  bits.
  - Space for **CS106BLECTURE**: 104 bits.
  - Space for **COPYRIGHTABLE**: 104 bits.
- **Question:** Can we reduce the number of bits needed to encode text?

# KIRK'S DIKDIK



# A Different Encoding

- ASCII uses one byte per character. There are 256 possible bytes.
- If we're specifically writing the string **KIRK'S DIKDIK**, which has only seven different characters, using full bytes is wasteful.
- Here's a three-bit encoding we can use to represent the letters in **KIRK'S DIKDIK**.
- This uses 37.5% as much space as what ASCII uses.

<i>character</i>	<i>code</i>
K	000
I	001
R	010
'	011
S	100
„	101
D	110

000	001	010	000	011	100	101	110	001	000	110	001	000
K	I	R	K	'	S	„	D	I	K	D	I	K

# Where We're Going

- Storing data using the ASCII encoding is portable across systems, but is not ideal in terms of space usage.
- Building custom codes for specific strings might let us save space.
- ***Idea:*** Use this approach to build a ***compression algorithm*** to reduce the amount of space needed to store text.

# The Key Idea

- If we can find a way to give all characters a bit pattern, that both the sender and receiver know about, and that can be decoded uniquely, then we can represent the same piece of text in multiple different ways.
- **Goal:** Find a way to do this that uses *less space* than the standard ASCII representation.

# Exploiting Redundancy

- Not all letters have the same frequency in **KIRK'S DIKDIK**.
- Here's the frequencies of each letter.
- So far, we've given each letter codes of the same length.
- **Key Question:** Can we give shorter encodings to more common characters?

<i>character</i>	<i>frequency</i>
K	4
I	3
D	2
R	1
'	1
S	1
U	1

# A First Attempt

*character*      *code*

<b>K</b>	<b>0</b>
<b>I</b>	<b>1</b>
<b>D</b>	<b>00</b>
<b>R</b>	<b>01</b>
<b>'</b>	<b>10</b>
<b>S</b>	<b>11</b>
<b>„</b>	<b>100</b>

01010101110000100010

0	1	01	0	10	11	100	00	1	0	00	1	0
<b>K</b>	<b>I</b>	<b>R</b>	<b>K</b>	<b>'</b>	<b>S</b>	<b>„</b>	<b>D</b>	<b>I</b>	<b>K</b>	<b>D</b>	<b>I</b>	<b>K</b>

# A First Attempt

*character*      *code*

<b>K</b>	<b>0</b>
<b>I</b>	<b>1</b>
<b>D</b>	<b>00</b>
<b>R</b>	<b>01</b>
<b>'</b>	<b>10</b>
<b>S</b>	<b>11</b>
<b>L</b>	<b>100</b>

01010101110000100010

# A First Attempt

*character*      *code*

<b>K</b>	<b>0</b>
<b>I</b>	<b>1</b>
<b>D</b>	<b>00</b>
<b>R</b>	<b>01</b>
<b>'</b>	<b>10</b>
<b>S</b>	<b>11</b>
<b>„</b>	<b>100</b>

01010101110000100010

0	1	01	0	10	11	100	00	1	0	00	1	0
<b>K</b>	<b>I</b>	<b>R</b>	<b>K</b>	<b>'</b>	<b>S</b>	<b>„</b>	<b>D</b>	<b>I</b>	<b>K</b>	<b>D</b>	<b>I</b>	<b>K</b>

# A First Attempt

*character*      *code*

<b>K</b>	<b>0</b>
<b>I</b>	<b>1</b>
<b>D</b>	<b>00</b>
<b>R</b>	<b>01</b>
<b>'</b>	<b>10</b>
<b>S</b>	<b>11</b>
<b>L</b>	<b>100</b>

01010101110000100010

01	01	01	01	1	10	0	00	10	0	0	10
R	R	R	R	I	'	K	D	'	K	K	'

# A First Attempt

*character*      *code*

<b>K</b>	<b>0</b>
<b>I</b>	<b>1</b>
<b>D</b>	<b>00</b>
<b>R</b>	<b>01</b>
<b>'</b>	<b>10</b>
<b>S</b>	<b>11</b>
<b>L</b>	<b>100</b>

01010101110



<b>01</b>	<b>01</b>	<b>01</b>	<b>01</b>	<b>1</b>	<b>10</b>	<b>0</b>	<b>00</b>	<b>10</b>	<b>0</b>	<b>0</b>	<b>10</b>
<b>R</b>	<b>R</b>	<b>R</b>	<b>R</b>	<b>I</b>	<b>'</b>	<b>K</b>	<b>D</b>	<b>'</b>	<b>K</b>	<b>K</b>	<b>'</b>

# The Problem

- If we use a different number of bits for each letter, we can't necessarily uniquely determine the boundaries between letters.
- We need an encoding that makes it possible to determine where one character stops and the next starts.
- Is this possible? If so, how?

# Prefix-Free Codes

- A ***prefix-free code*** is an encoding system in which no code is a prefix of another code.
- Here's a sample prefix code for the letters in **KIRK'S DIKDIK**.

<i>character</i>	<i>code</i>
K	10
I	01
D	111
R	001
'	000
S	1101
U	1100

# Prefix-Free Codes

*character      code*

<b>K</b>	<b>10</b>
<b>I</b>	<b>01</b>
<b>D</b>	<b>111</b>
<b>R</b>	<b>001</b>
<b>'</b>	<b>000</b>
<b>S</b>	<b>1101</b>
<b>L</b>	<b>1100</b>

# Prefix-Free Codes

*character      code*

<b>K</b>	<b>10</b>
<b>I</b>	<b>01</b>
<b>D</b>	<b>111</b>
<b>R</b>	<b>001</b>
<b>'</b>	<b>000</b>
<b>S</b>	<b>1101</b>
<b>„</b>	<b>1100</b>

<b>10</b>	<b>01</b>	<b>001</b>	<b>10</b>	<b>000</b>	<b>1101</b>	<b>1100</b>	<b>111</b>	<b>01</b>	<b>10</b>	<b>111</b>	<b>01</b>	<b>10</b>
<b>K</b>	<b>I</b>	<b>R</b>	<b>K</b>	<b>'</b>	<b>S</b>	<b>„</b>	<b>D</b>	<b>I</b>	<b>K</b>	<b>D</b>	<b>I</b>	<b>K</b>

# Prefix-Free Codes

*character      code*

<b>K</b>	<b>10</b>
<b>I</b>	<b>01</b>
<b>D</b>	<b>111</b>
<b>R</b>	<b>001</b>
<b>'</b>	<b>000</b>
<b>S</b>	<b>1101</b>
<b>„</b>	<b>1100</b>

1001001100001101110011101101110110

<b>10</b>	<b>01</b>	<b>001</b>	<b>10</b>	<b>000</b>	<b>1101</b>	<b>1100</b>	<b>111</b>	<b>01</b>	<b>10</b>	<b>111</b>	<b>01</b>	<b>10</b>
<b>K</b>	<b>I</b>	<b>R</b>	<b>K</b>	<b>'</b>	<b>S</b>	<b>„</b>	<b>D</b>	<b>I</b>	<b>K</b>	<b>D</b>	<b>I</b>	<b>K</b>

# Prefix-Free Codes

*character*      *code*

<b>K</b>	<b>10</b>
<b>I</b>	<b>01</b>
<b>D</b>	<b>111</b>
<b>R</b>	<b>001</b>
<b>'</b>	<b>000</b>
<b>S</b>	<b>1101</b>
<b>L</b>	<b>1100</b>

1001001100001101110011101101110110

# Prefix-Free Codes

*character*      *code*

<b>K</b>	<b>10</b>
<b>I</b>	<b>01</b>
<b>D</b>	<b>111</b>
<b>R</b>	<b>001</b>
<b>'</b>	<b>000</b>
<b>S</b>	<b>1101</b>
<b>L</b>	<b>1100</b>

1001001100001101110011101101110110

# Prefix-Free Codes

*character*      *code*

<b>K</b>	<b>10</b>
<b>I</b>	<b>01</b>
<b>D</b>	<b>111</b>
<b>R</b>	<b>001</b>
<b>'</b>	<b>000</b>
<b>S</b>	<b>1101</b>
<b>L</b>	<b>1100</b>

1001001100001101110011101101110110

# Prefix-Free Codes

*character*      *code*

<b>K</b>	<b>10</b>
<b>I</b>	<b>01</b>
<b>D</b>	<b>111</b>
<b>R</b>	<b>001</b>
<b>'</b>	<b>000</b>
<b>S</b>	<b>1101</b>
<b>L</b>	<b>1100</b>

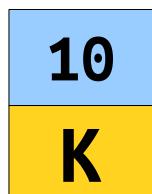
**1001001100001101110011101101110110**

# Prefix-Free Codes

*character*      *code*

K	10
I	01
D	111
R	001
'	000
S	1101
L	1100

1001001100001101110011101101110110

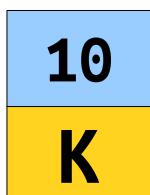


# Prefix-Free Codes

*character*      *code*

<b>K</b>	<b>10</b>
<b>I</b>	<b>01</b>
<b>D</b>	<b>111</b>
<b>R</b>	<b>001</b>
<b>'</b>	<b>000</b>
<b>S</b>	<b>1101</b>
<b>L</b>	<b>1100</b>

1001001100001101110011101101110110

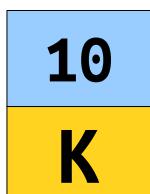


# Prefix-Free Codes

*character*      *code*

<b>K</b>	<b>10</b>
<b>I</b>	<b>01</b>
<b>D</b>	<b>111</b>
<b>R</b>	<b>001</b>
<b>'</b>	<b>000</b>
<b>S</b>	<b>1101</b>
<b>L</b>	<b>1100</b>

1001001100001101110011101101101110110

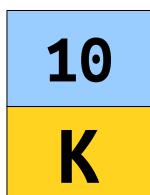


# Prefix-Free Codes

*character*      *code*

K	10
I	01
D	111
R	001
'	000
S	1101
L	1100

1001001100001101110011101101101110110



# Prefix-Free Codes

*character*      *code*

K	10
I	01
D	111
R	001
'	000
S	1101
L	1100

1001001100001101110011101101101110110

10	01
K	I

# Prefix-Free Codes

*character*      *code*

<b>K</b>	<b>10</b>
<b>I</b>	<b>01</b>
<b>D</b>	<b>111</b>
<b>R</b>	<b>001</b>
<b>'</b>	<b>000</b>
<b>S</b>	<b>1101</b>
<b>L</b>	<b>1100</b>

1001001100001101110011101101101110110

<b>10</b>	<b>01</b>
<b>K</b>	<b>I</b>

# Prefix-Free Codes

*character*      *code*

<b>K</b>	<b>10</b>
<b>I</b>	<b>01</b>
<b>D</b>	<b>111</b>
<b>R</b>	<b>001</b>
<b>'</b>	<b>000</b>
<b>S</b>	<b>1101</b>
<b>L</b>	<b>1100</b>

1001001100001101110011101101101110110

<b>10</b>	<b>01</b>
<b>K</b>	<b>I</b>

# Prefix-Free Codes

*character*      *code*

<b>K</b>	<b>10</b>
<b>I</b>	<b>01</b>
<b>D</b>	<b>111</b>
<b>R</b>	<b>001</b>
<b>'</b>	<b>000</b>
<b>S</b>	<b>1101</b>
<b>L</b>	<b>1100</b>

1001001100001101110011101101101110110

<b>10</b>	<b>01</b>
<b>K</b>	<b>I</b>

# Prefix-Free Codes

*character*      *code*

K	10
I	01
D	111
R	001
'	000
S	1101
L	1100

1001001100001101110011101101110110

10	01
K	I

# Prefix-Free Codes

*character*      *code*

K	10
I	01
D	111
R	001
'	000
S	1101
L	1100

1001001100001101110011101101110110

10	01	001
K	I	R

# Prefix-Free Codes

*character*      *code*

<b>K</b>	<b>10</b>
<b>I</b>	<b>01</b>
<b>D</b>	<b>111</b>
<b>R</b>	<b>001</b>
<b>'</b>	<b>000</b>
<b>S</b>	<b>1101</b>
<b>L</b>	<b>1100</b>

1001001100001101110011101101110110

<b>10</b>	<b>01</b>	<b>001</b>
<b>K</b>	<b>I</b>	<b>R</b>

# Prefix-Free Codes

- Using this prefix code, we can represent **KIRK'S DIKDIK** as the sequence

1001001100001101110011101101110110

- This uses just 34 bits, compared to our initial 104. Wow!
- But where did this code come from? How could you come up with codes like this for other strings?

<i>character</i>	<i>code</i>
K	10
I	01
D	111
R	001
'	000
S	1101
U	1100

1001001100001101110011101101110110

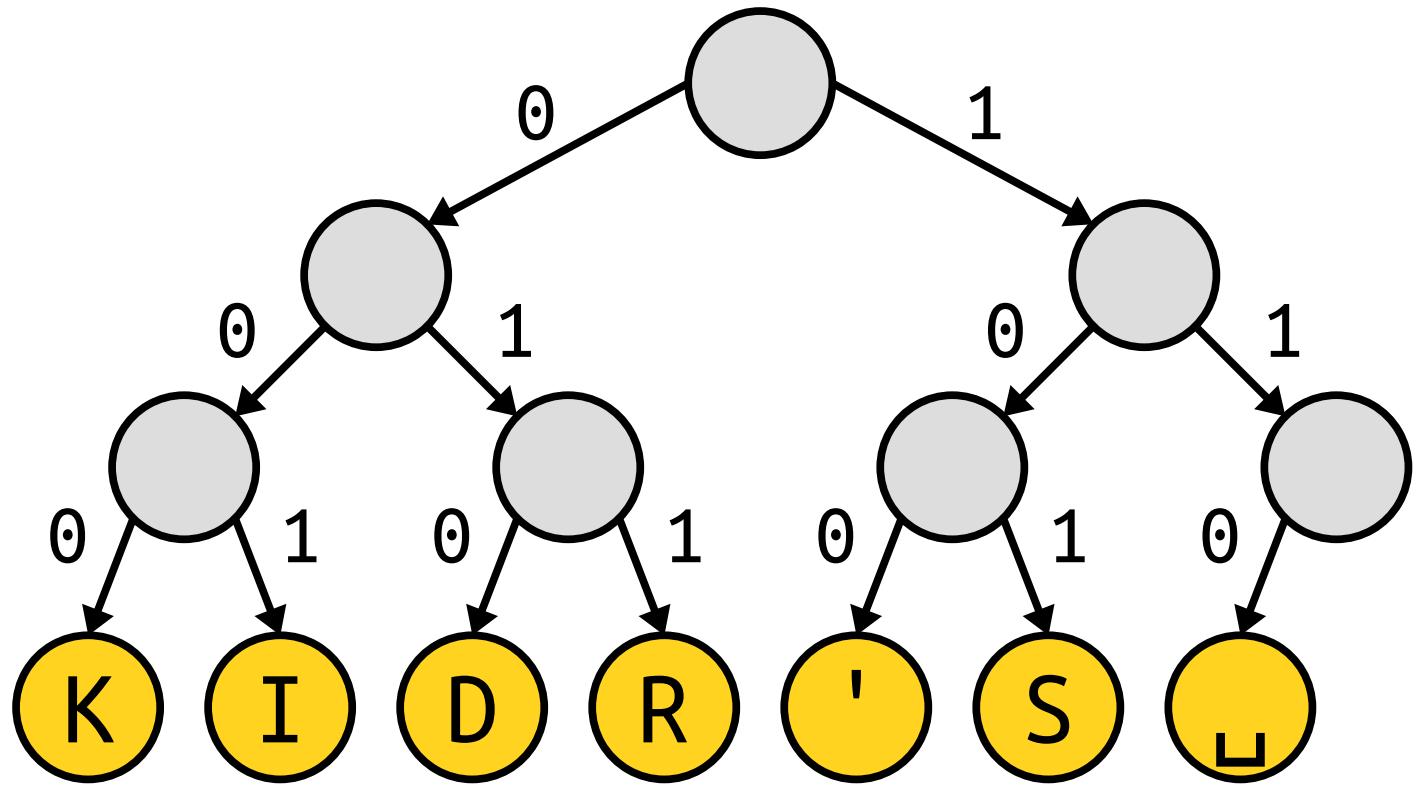
<i>character</i>	<i>code</i>
K	1111110
I	111110
D	11110
R	1110
'	110
S	10
U	0

11111101111101110111111011010011110111110111111011111011111101111110

How do you find a “good” prefix-free code?

# The Main Insight

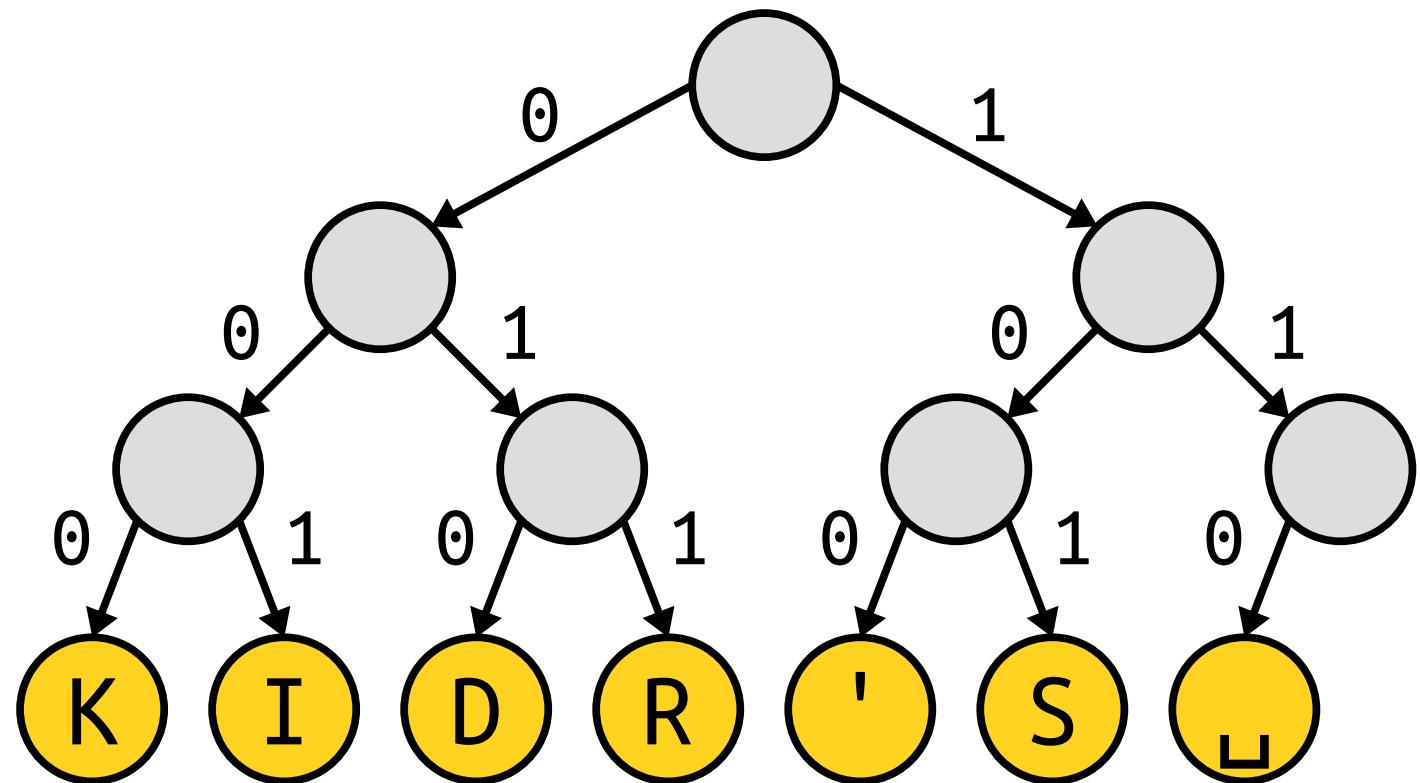
<i>character</i>	<i>code</i>
K	000
I	001
D	010
R	011
'	100
S	101
U	110

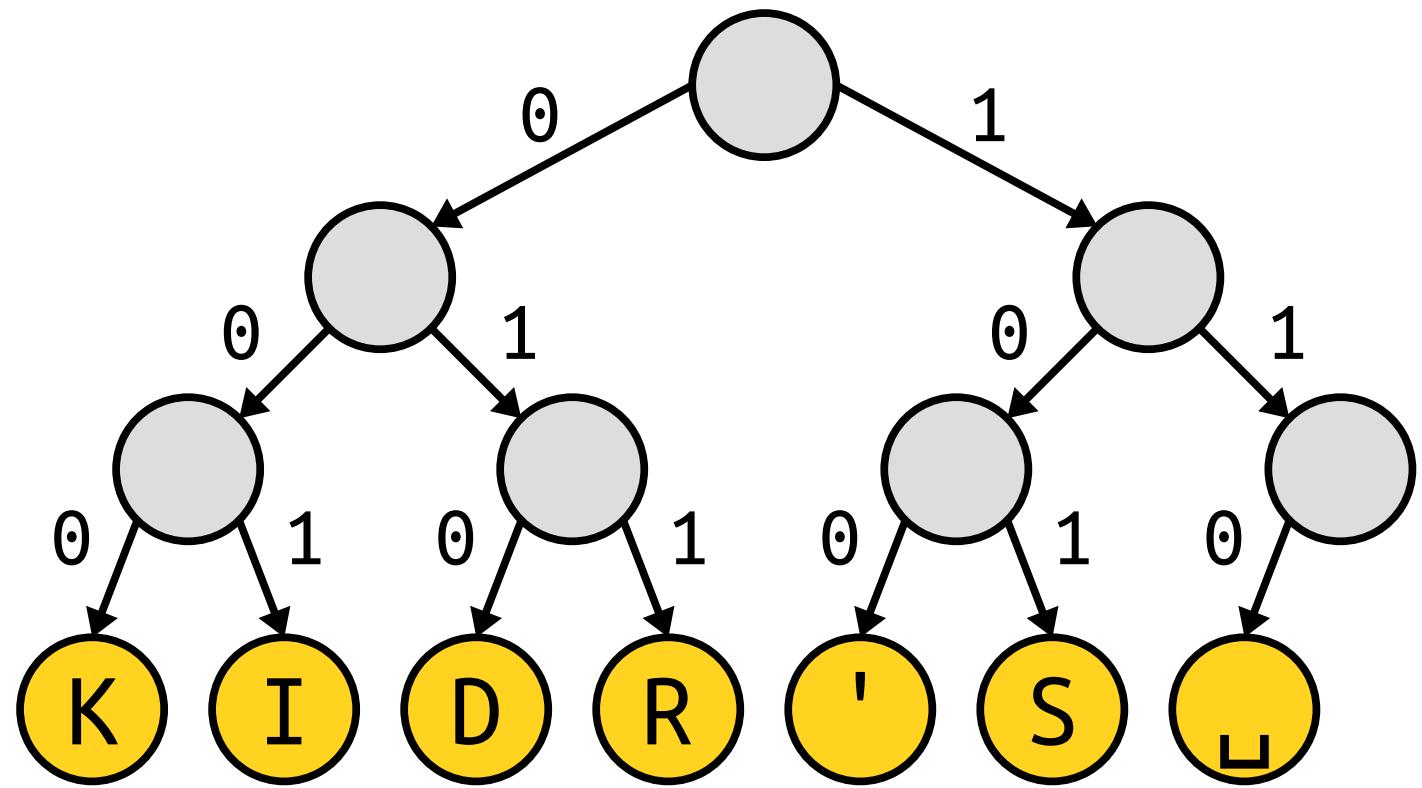


This special type of binary tree is called a ***coding tree***.

*character*      *code*

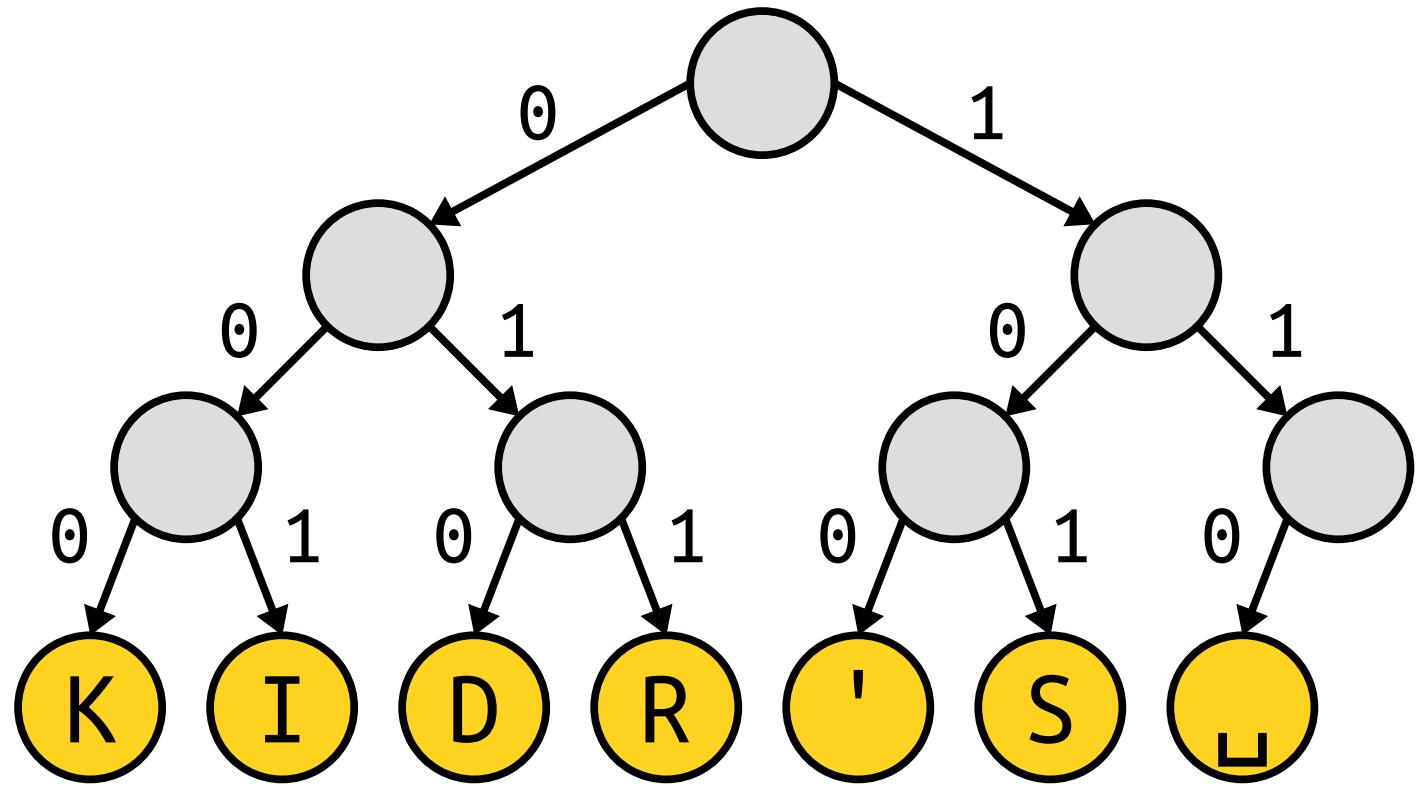
K	000
I	001
D	010
R	011
'	100
S	101
„	110





101000001

What is the title of this slide?

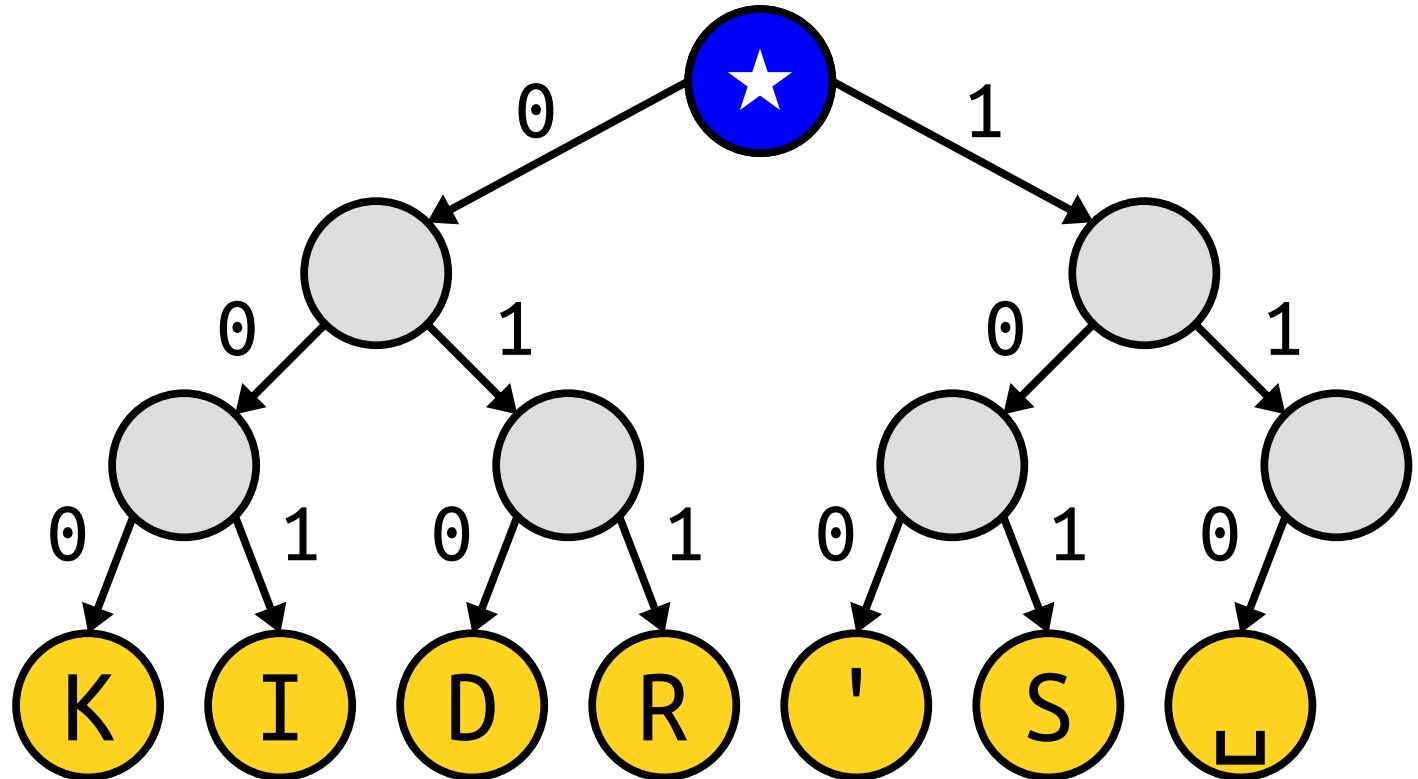


Answer at

<https://pollev.com/cs106bwin23>

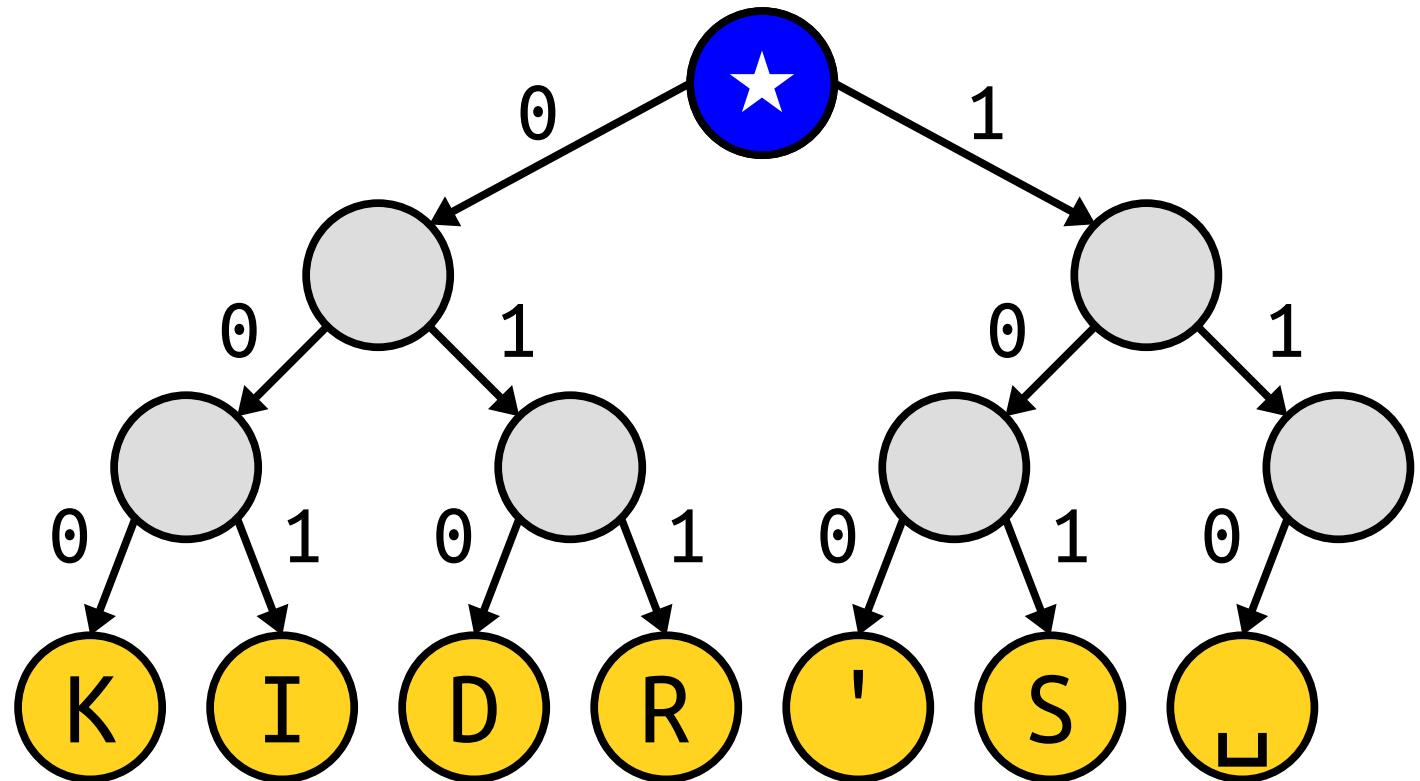
# 101000001

What is the title of this slide?



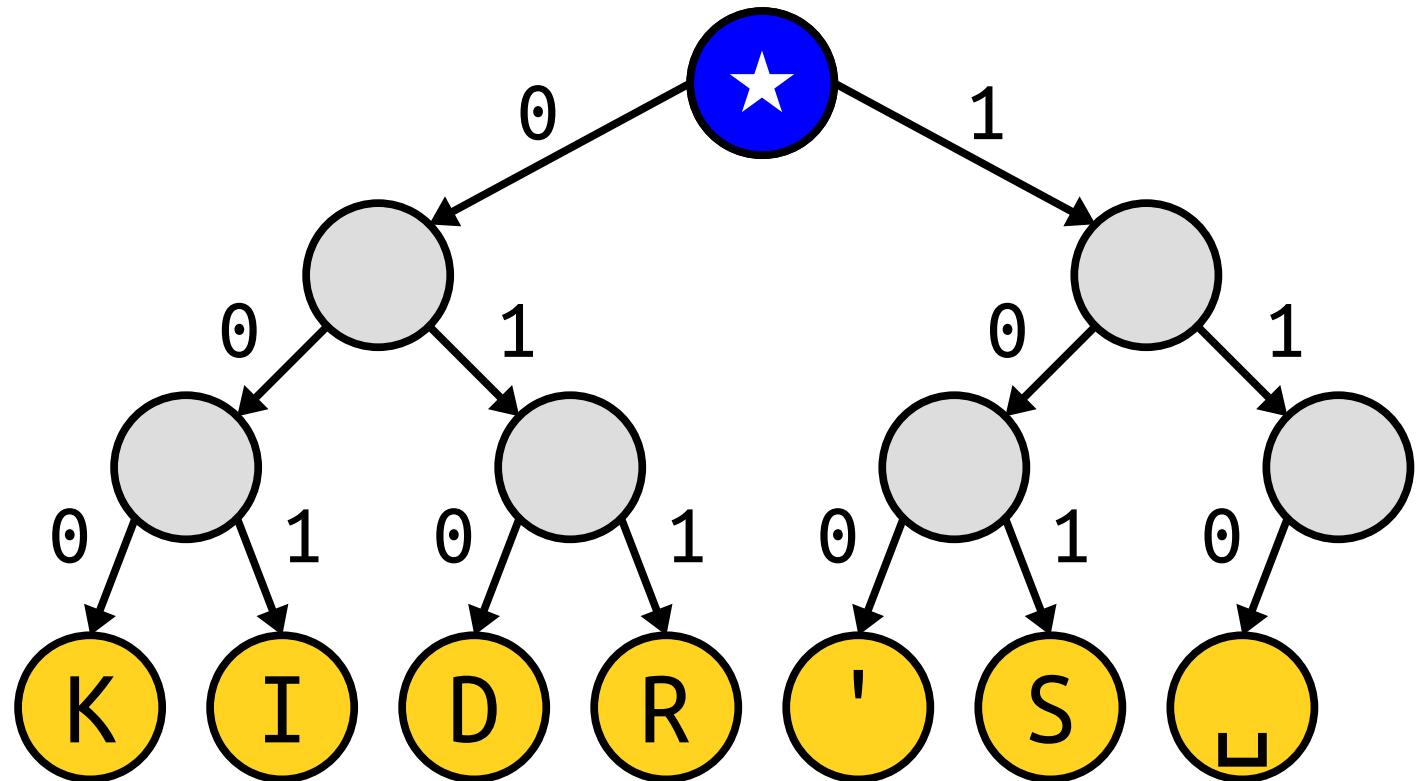
101000001

What is the title of this slide?



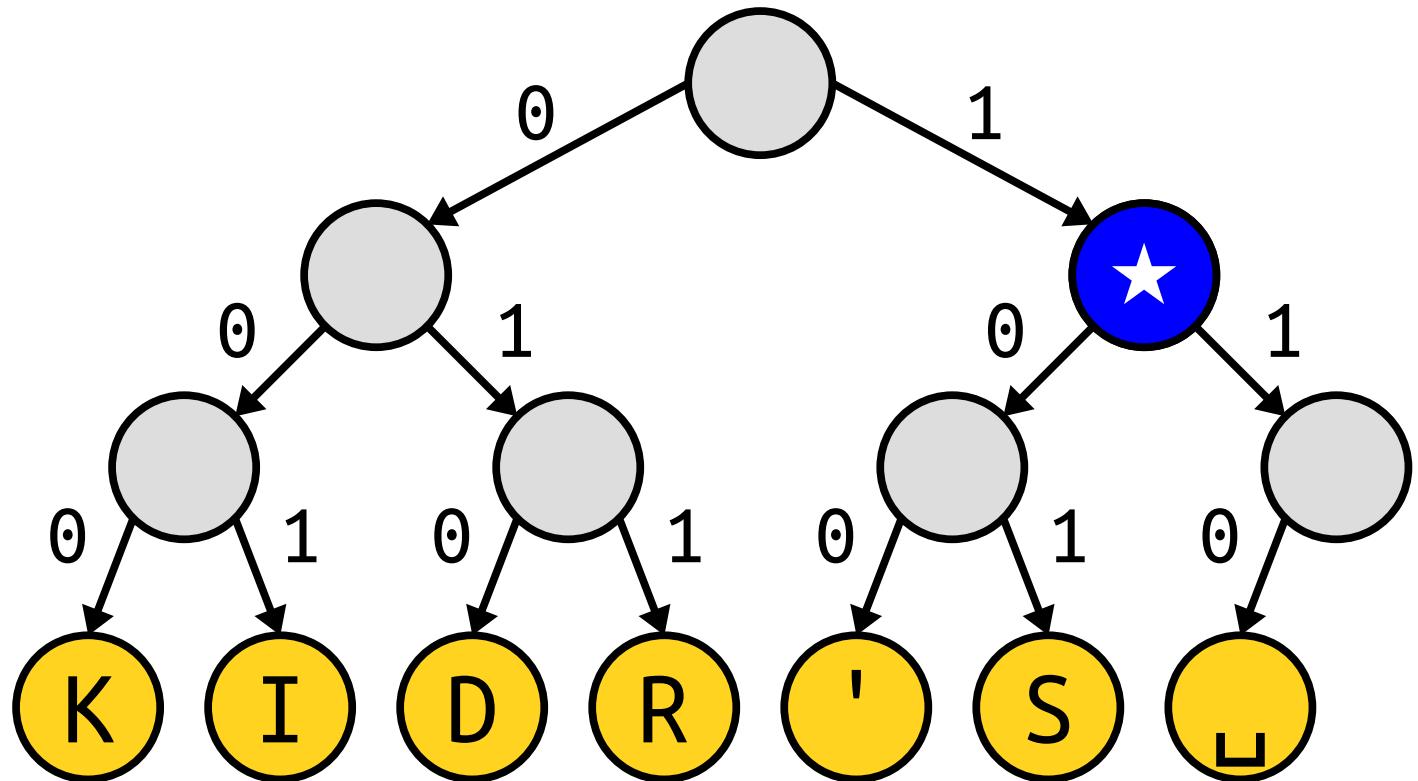
101000001

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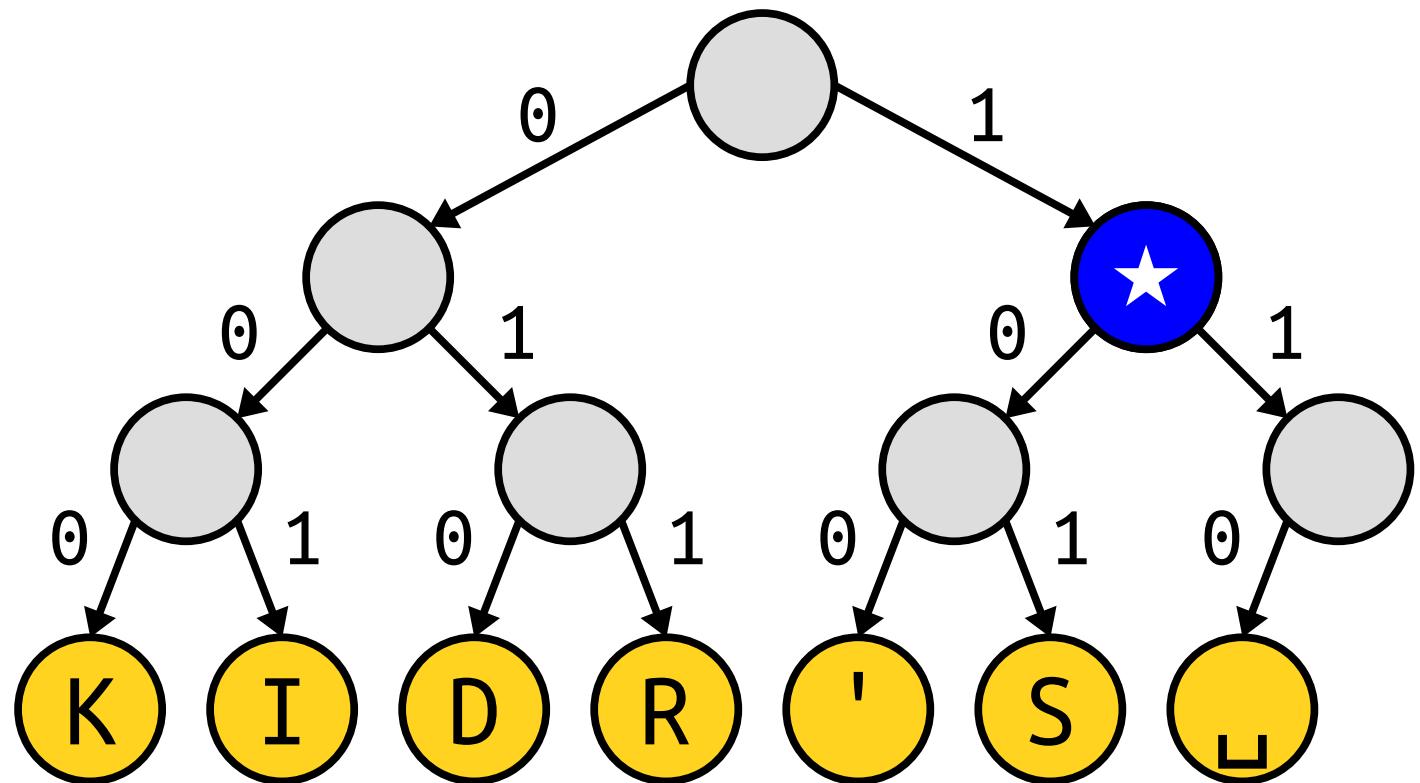
101000001

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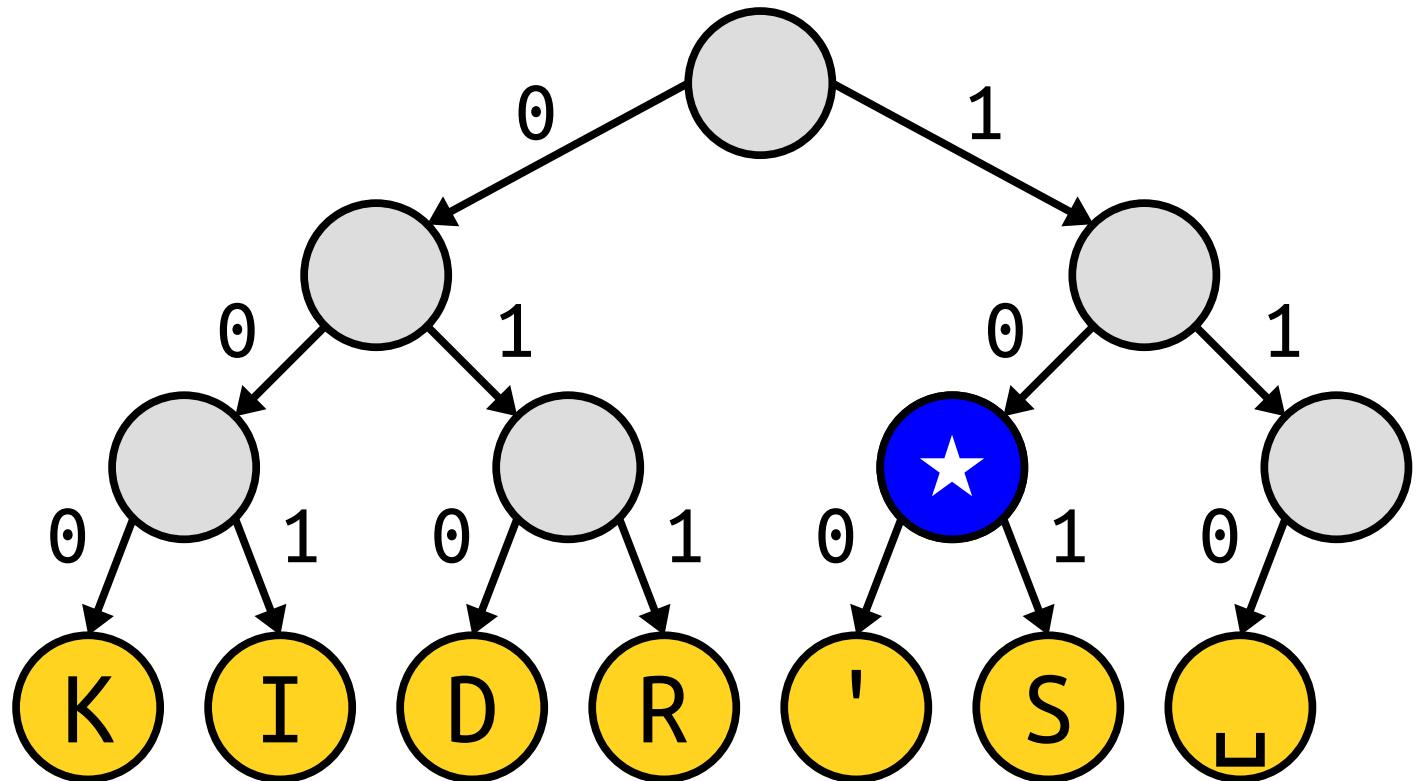
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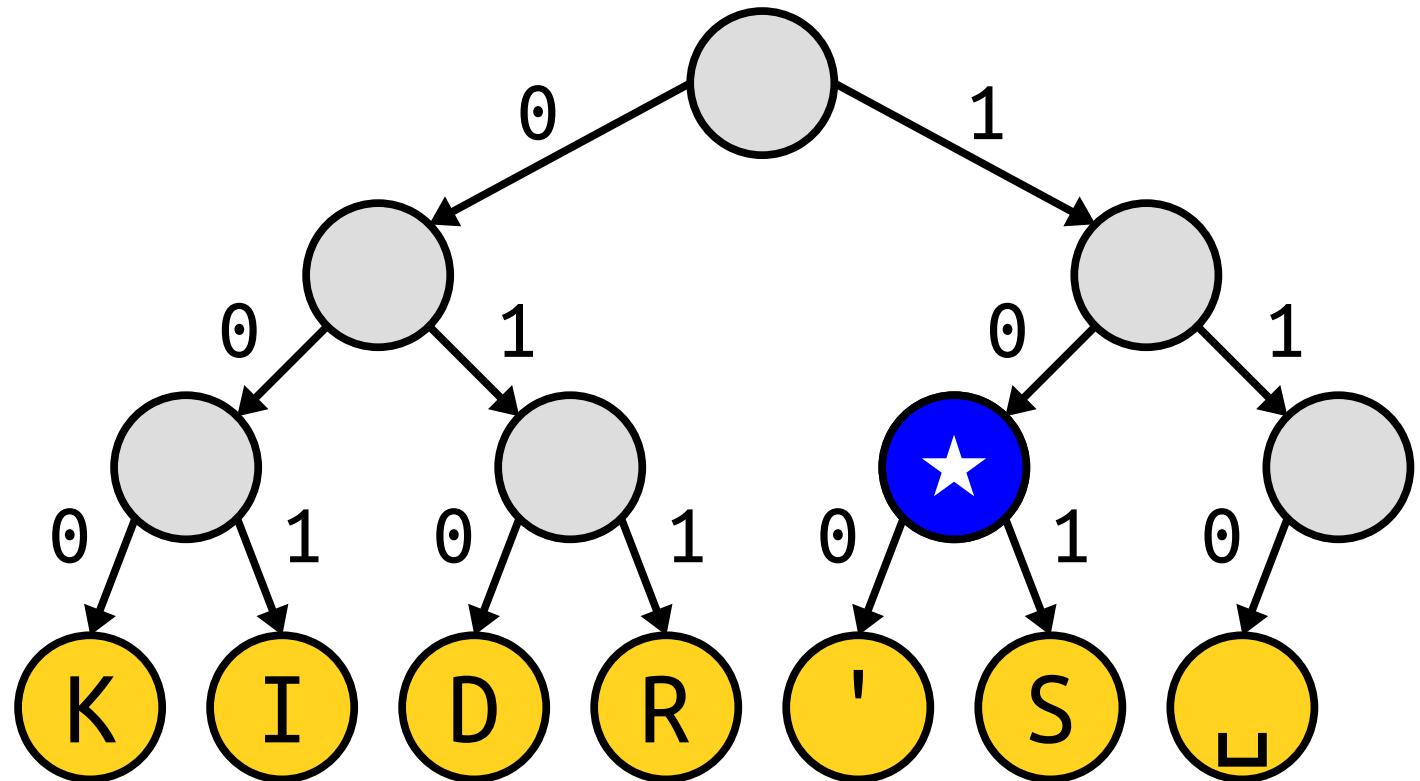
101000001

What is the title of this slide?



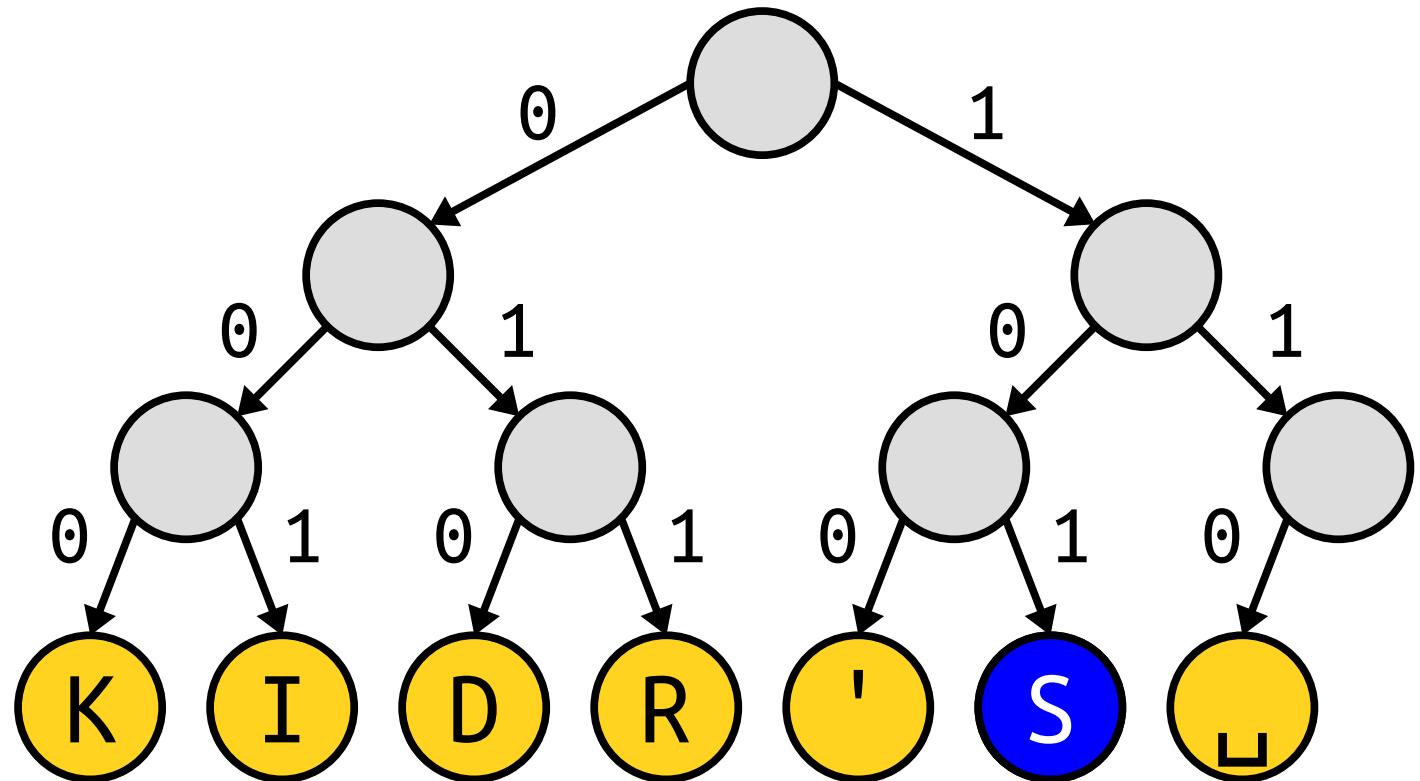
101000001

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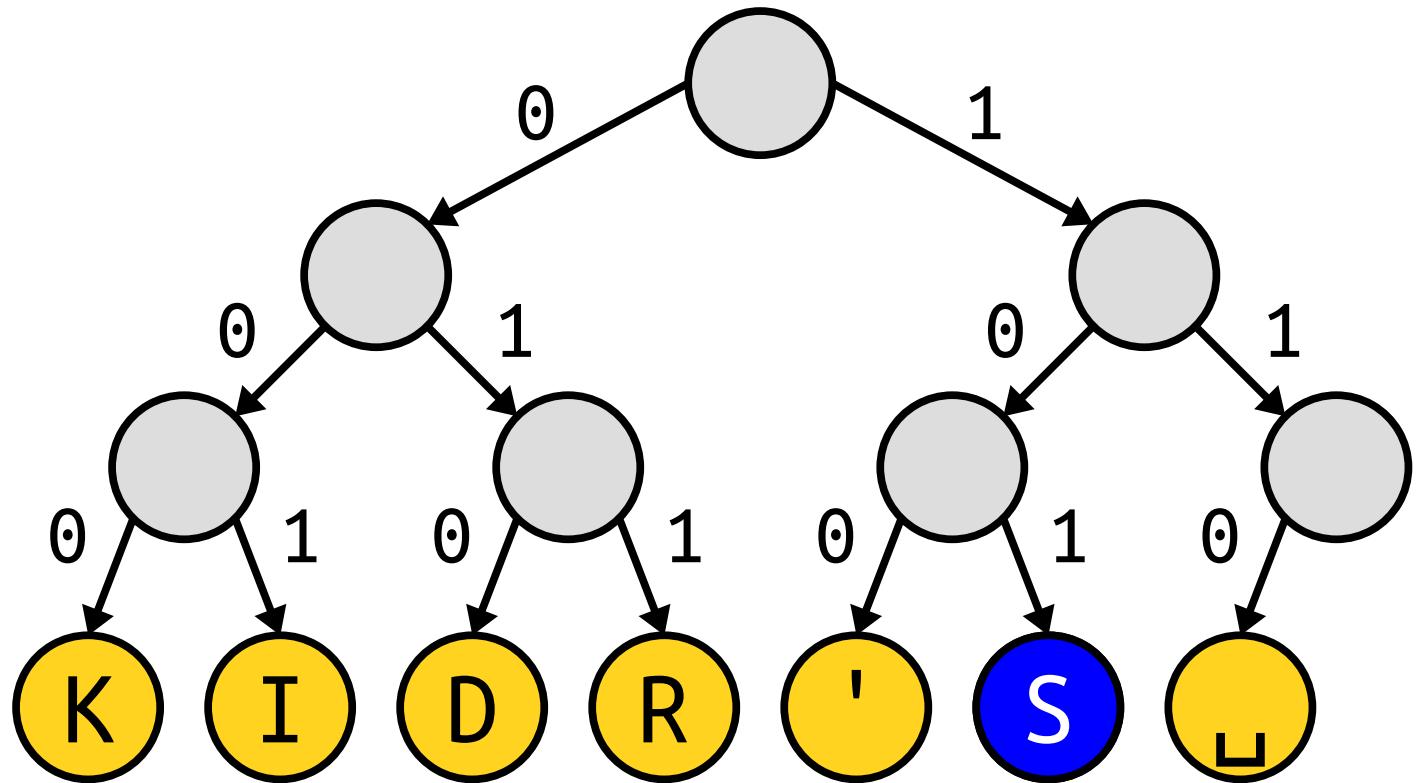
101000001

What is the title of this slide?



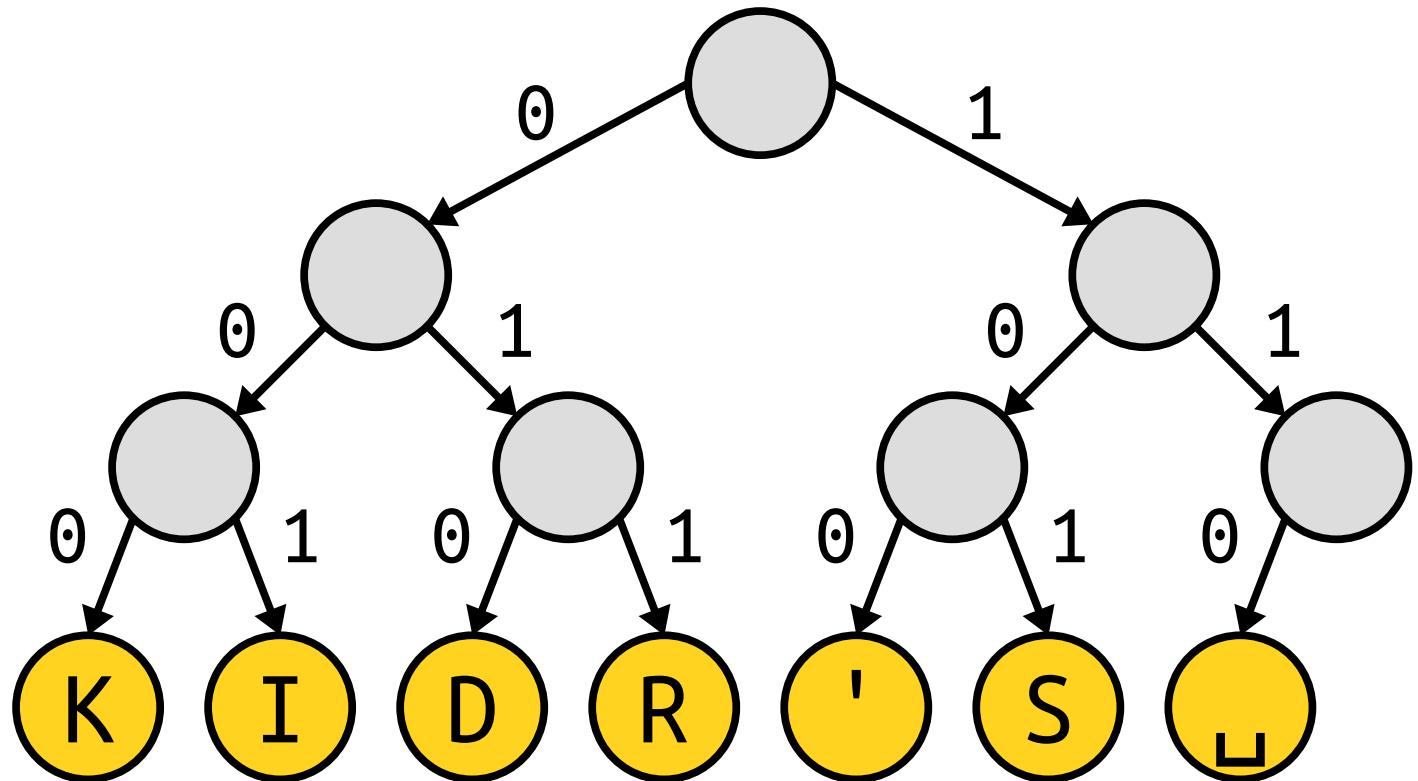
**S** 000001

What is the title of this slide?



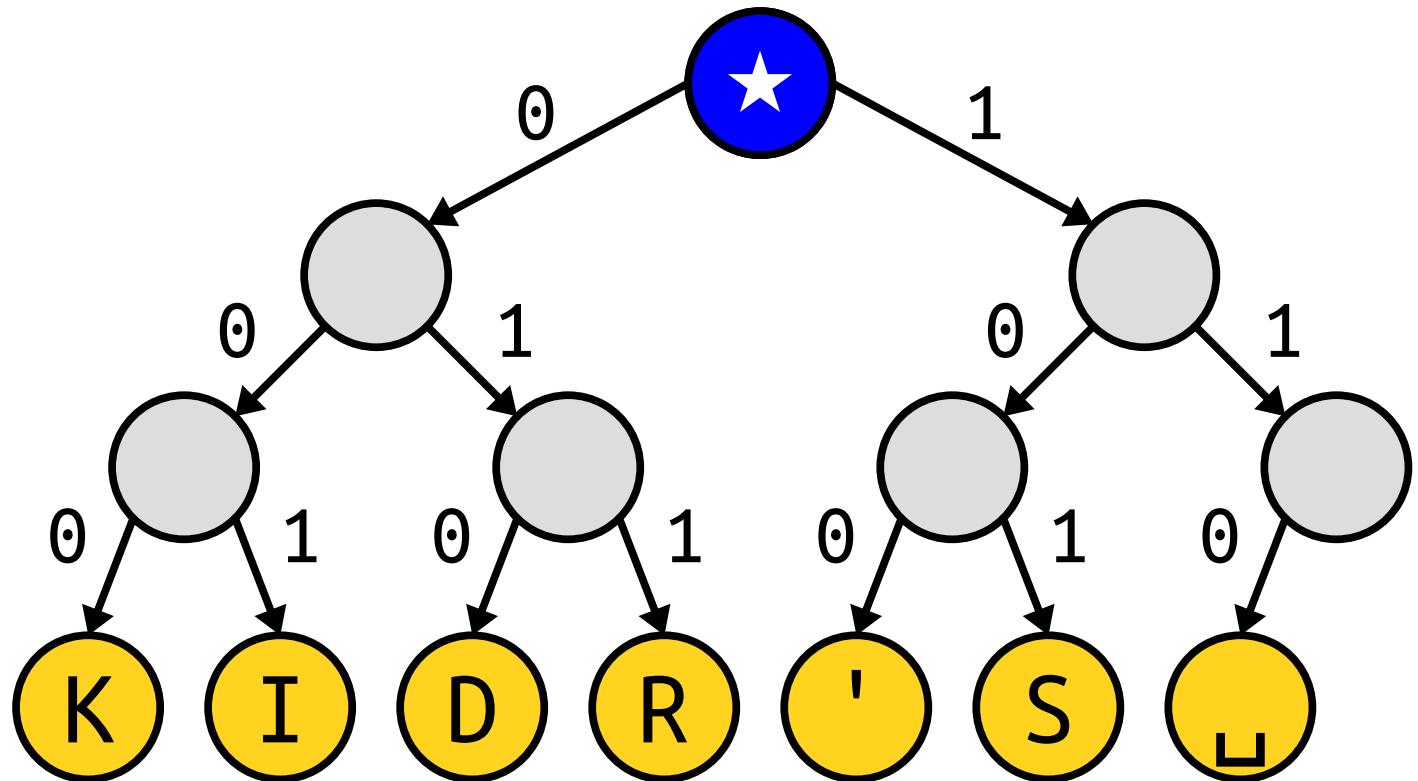
**S** 000001

What is the title of this slide?



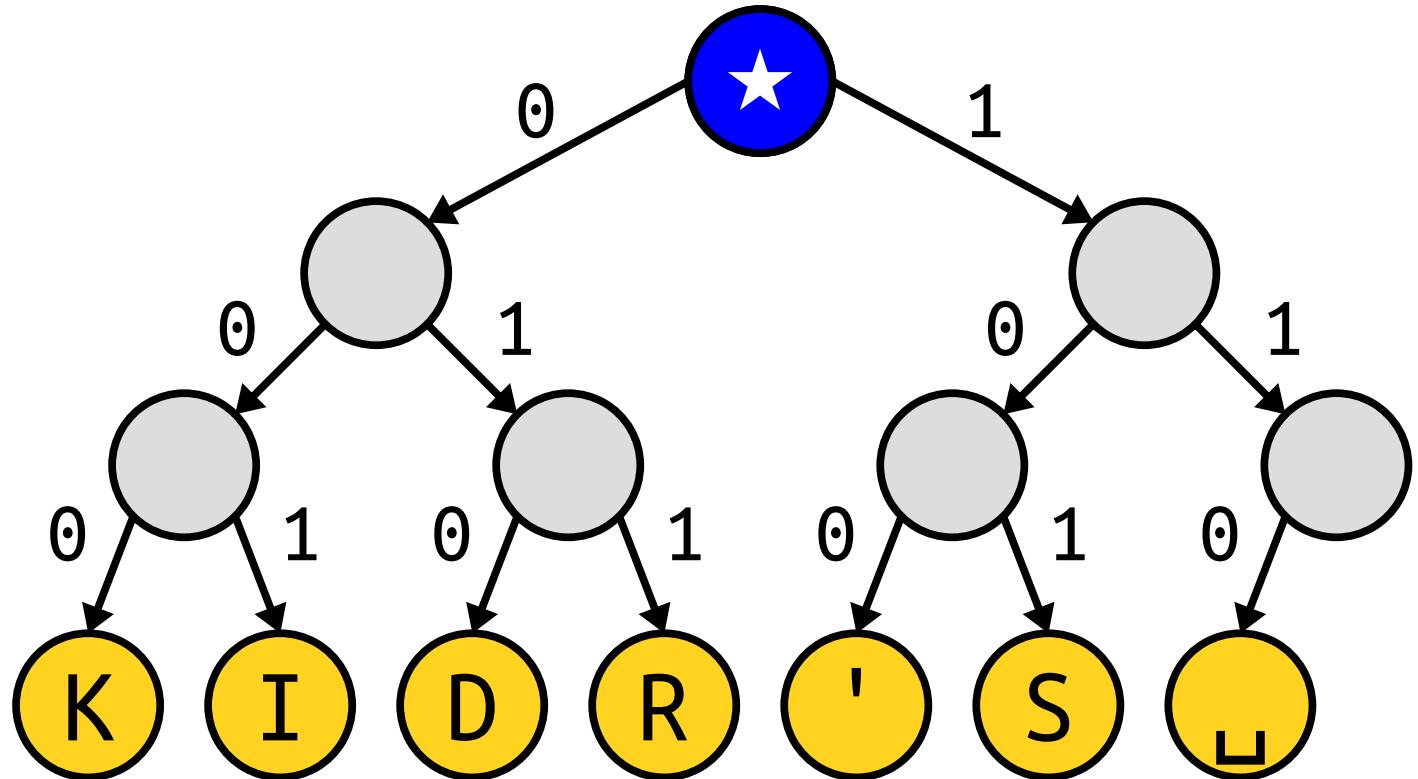
**S** 000001

What is the title of this slide?



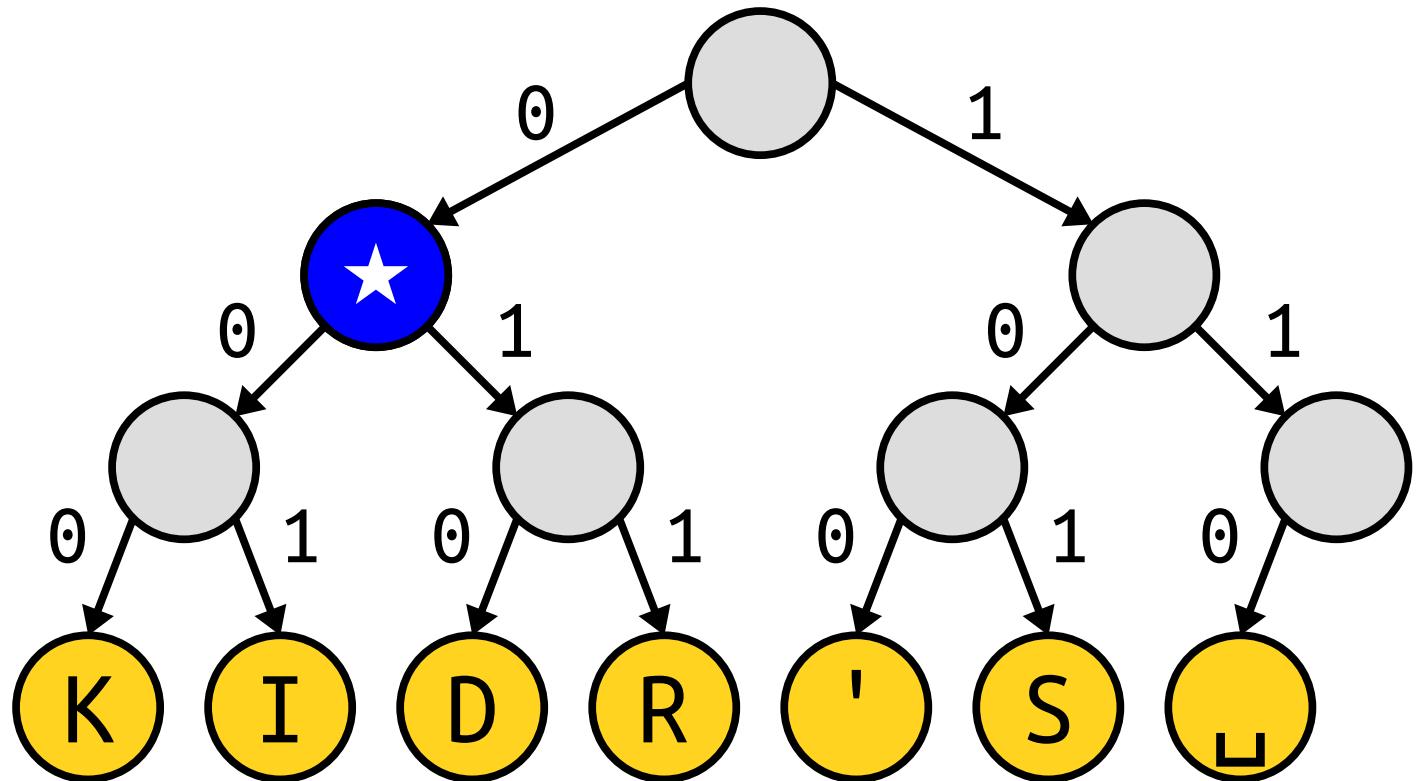
**S** 000001

What is the title of this slide?



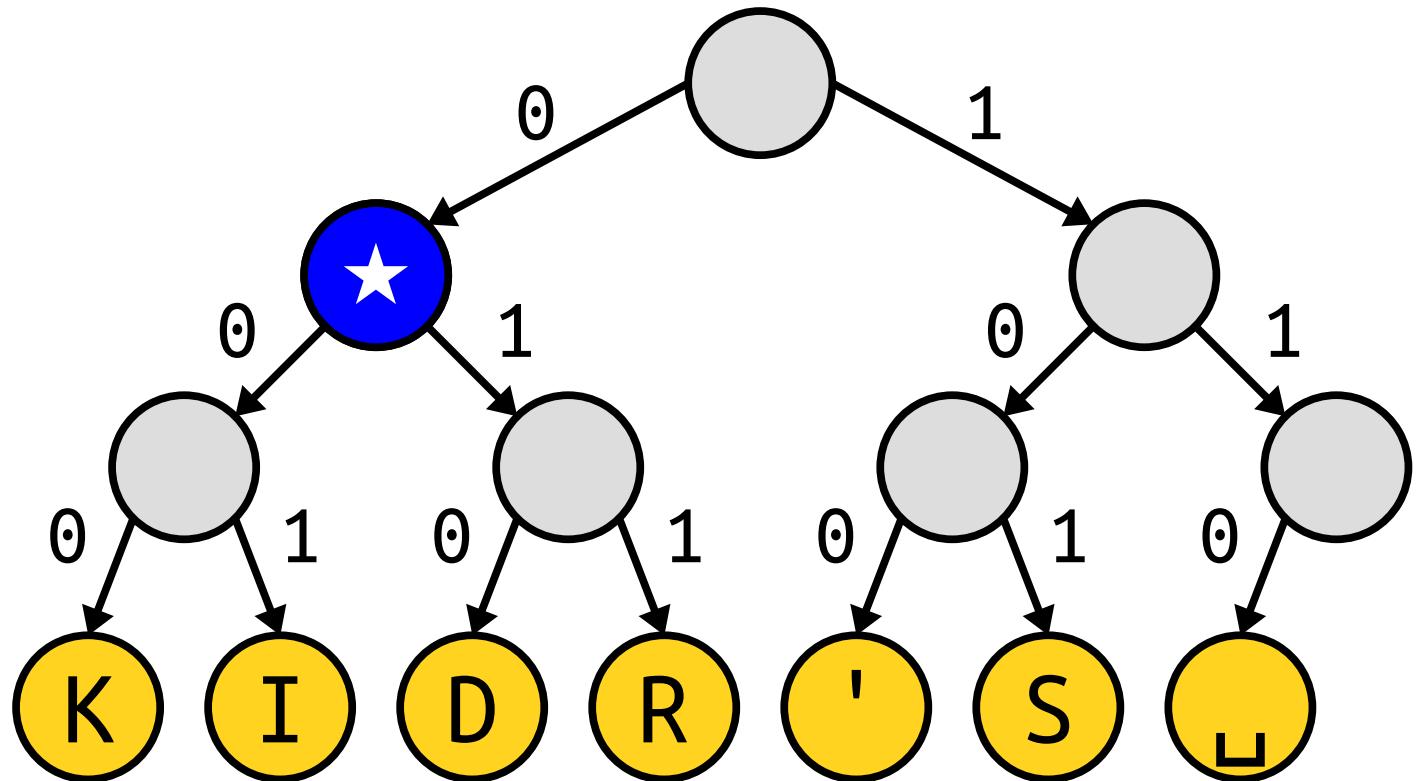
**S** 000001

What is the title of this slide?



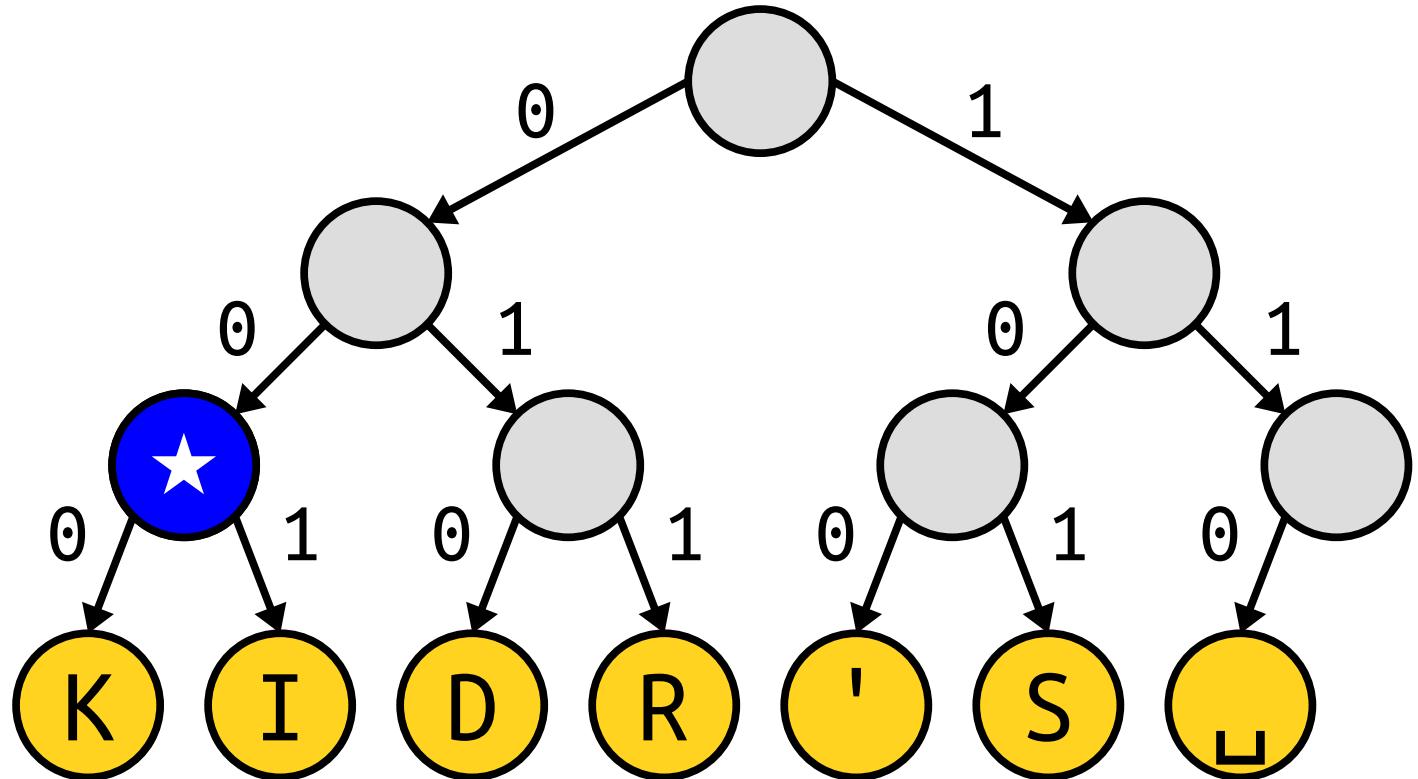
**S** 0000001

What is the title of this slide?



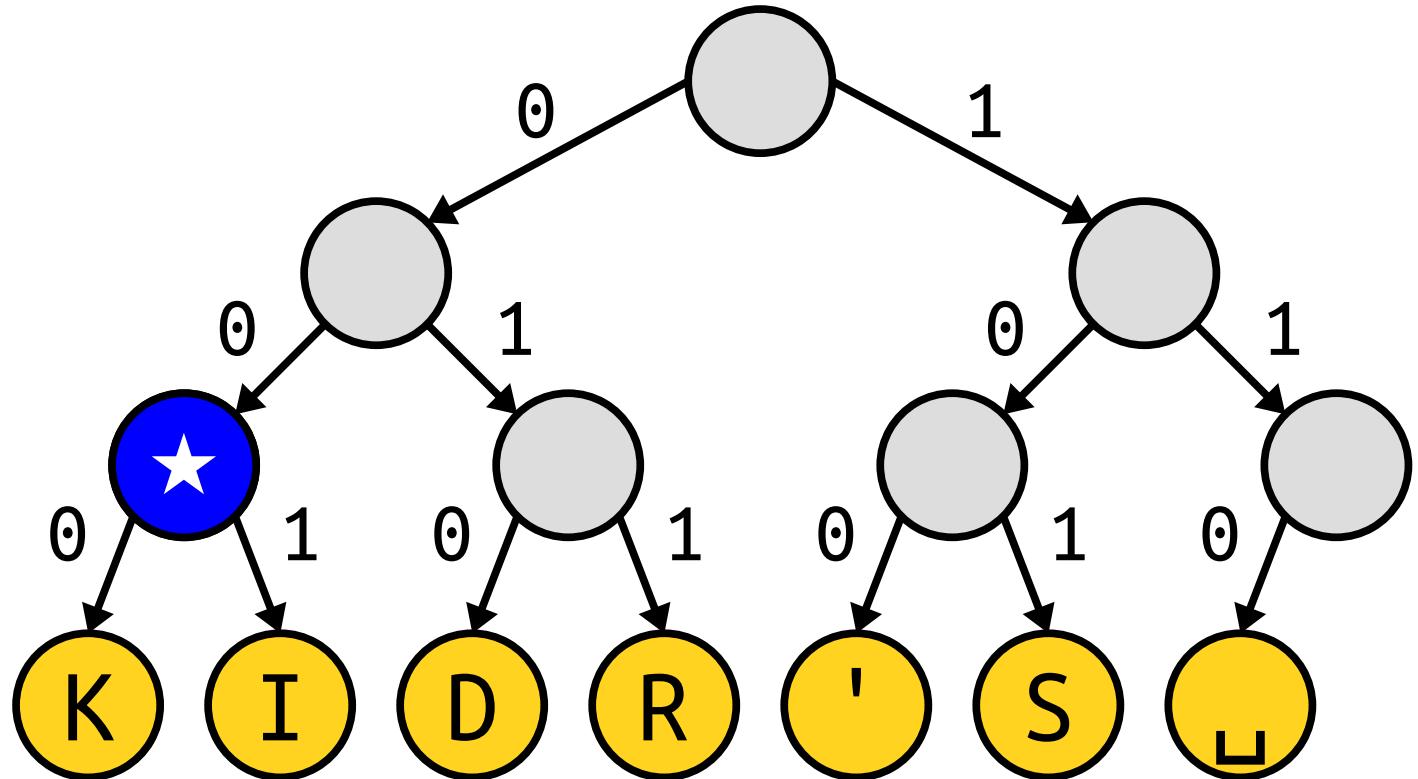
**S** 000001

What is the title of this slide?



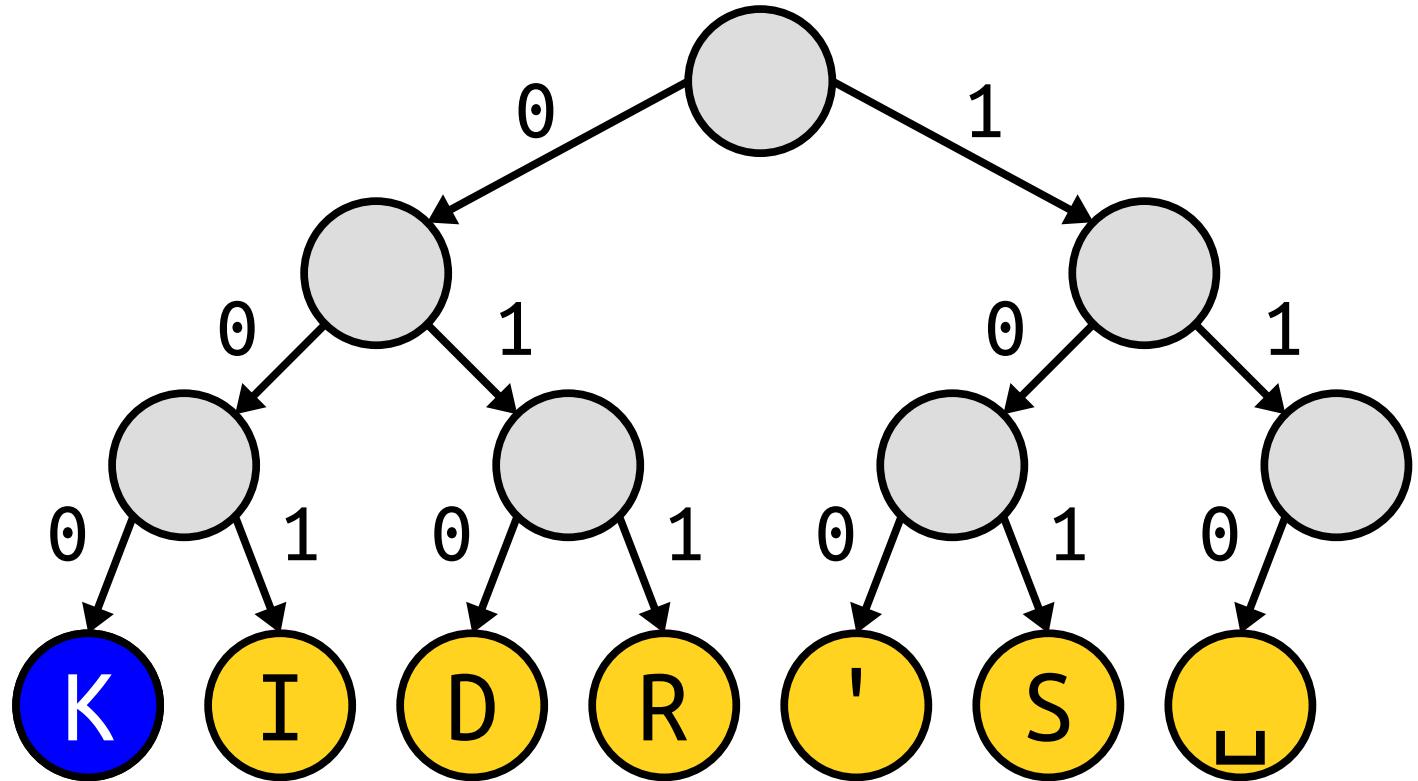
**S** 000001

What is the title of this slide?



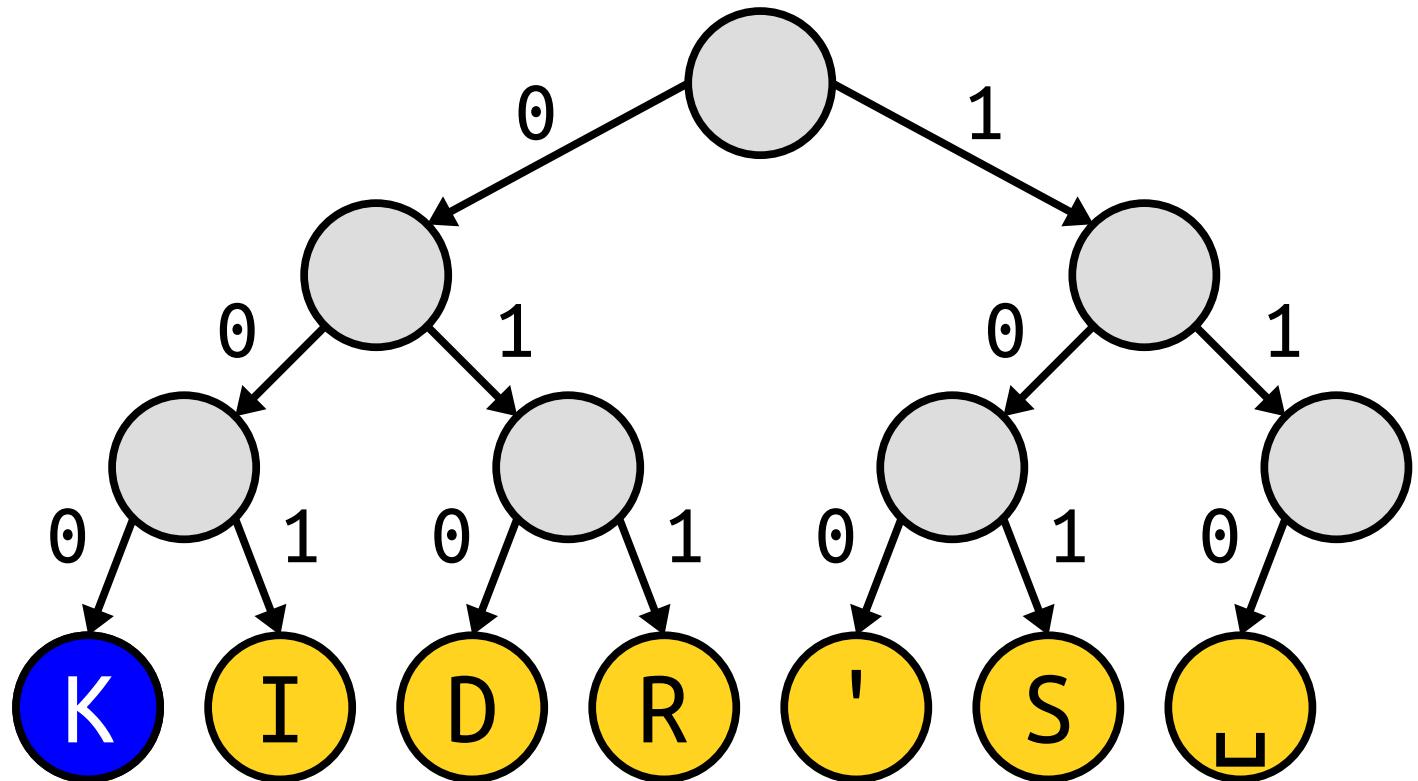
**S** 000001

What is the title of this slide?



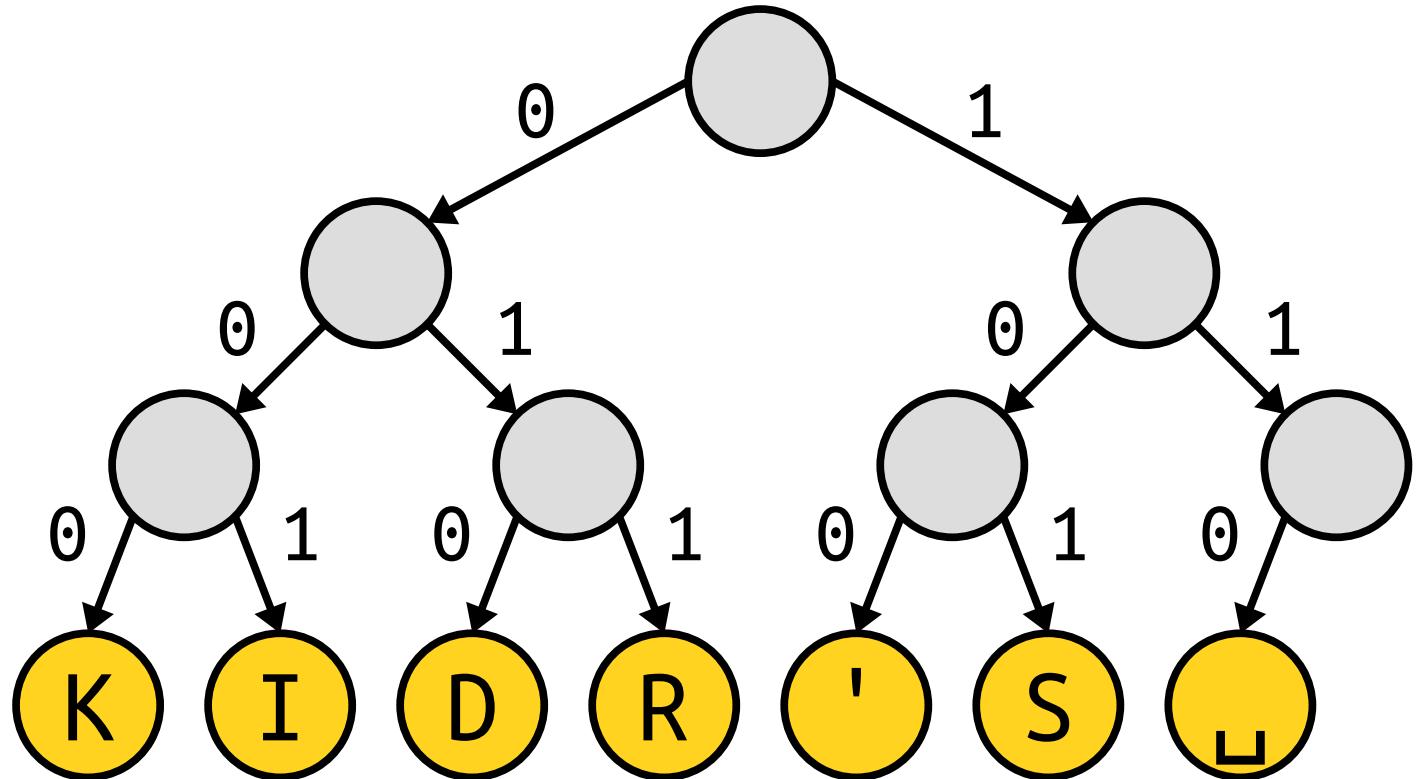
S K 001

What is the title of this slide?



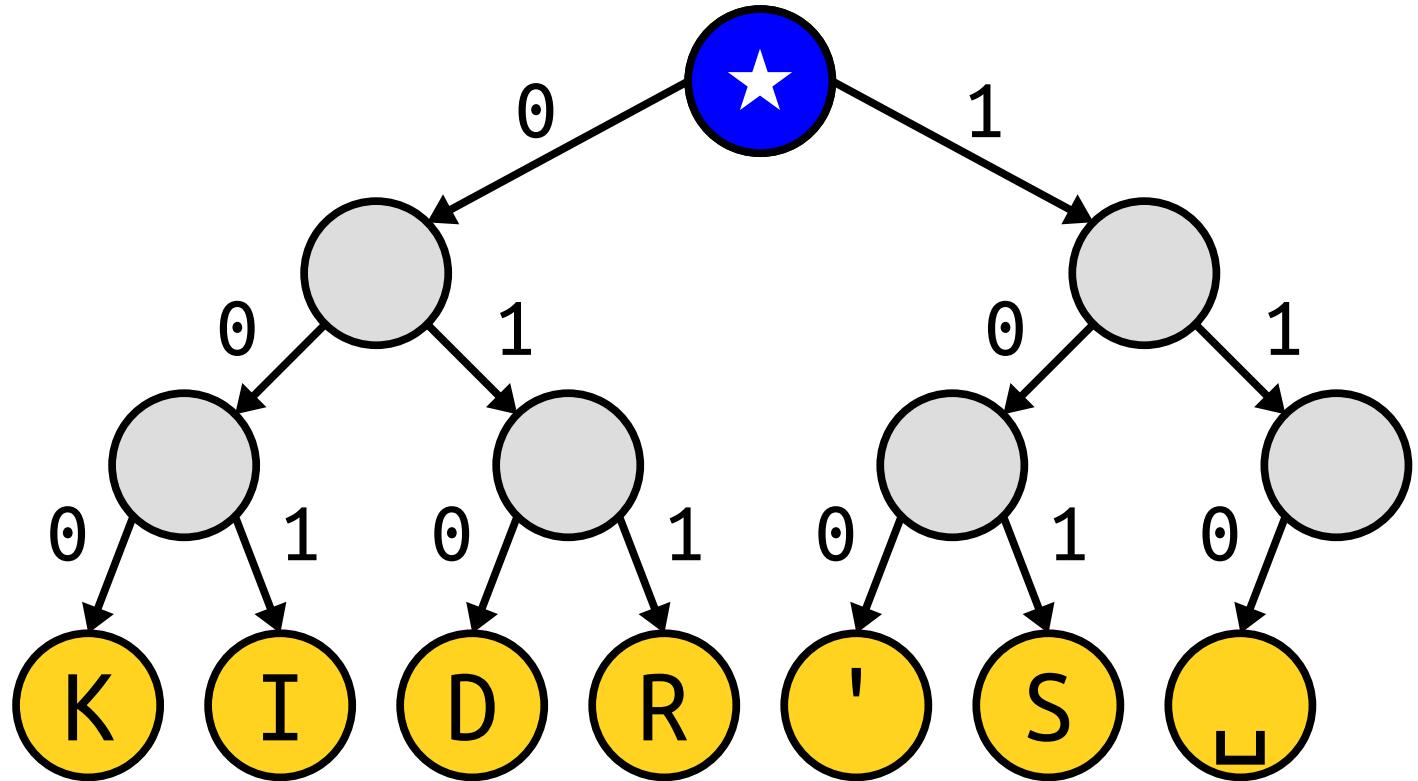
S K 001

What is the title of this slide?



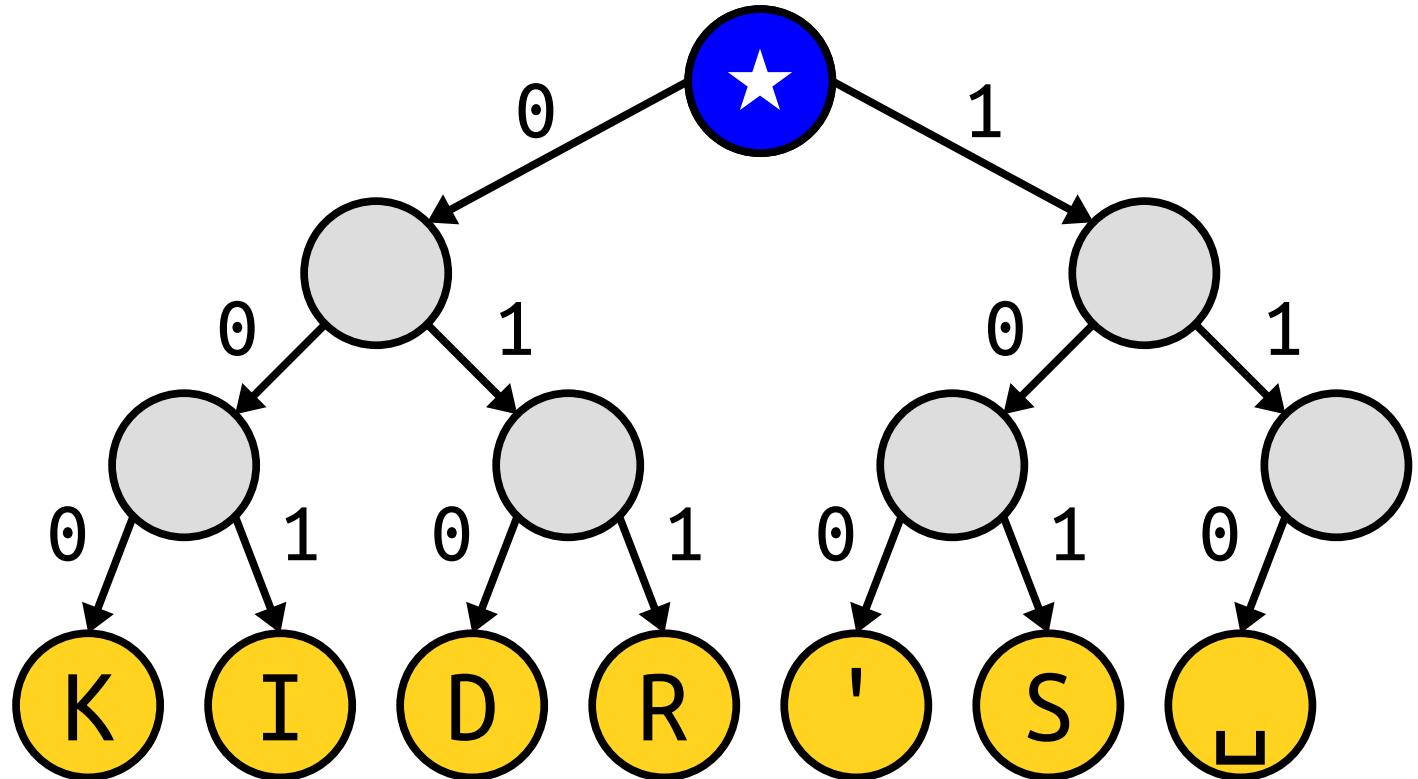
S K 001

What is the title of this slide?



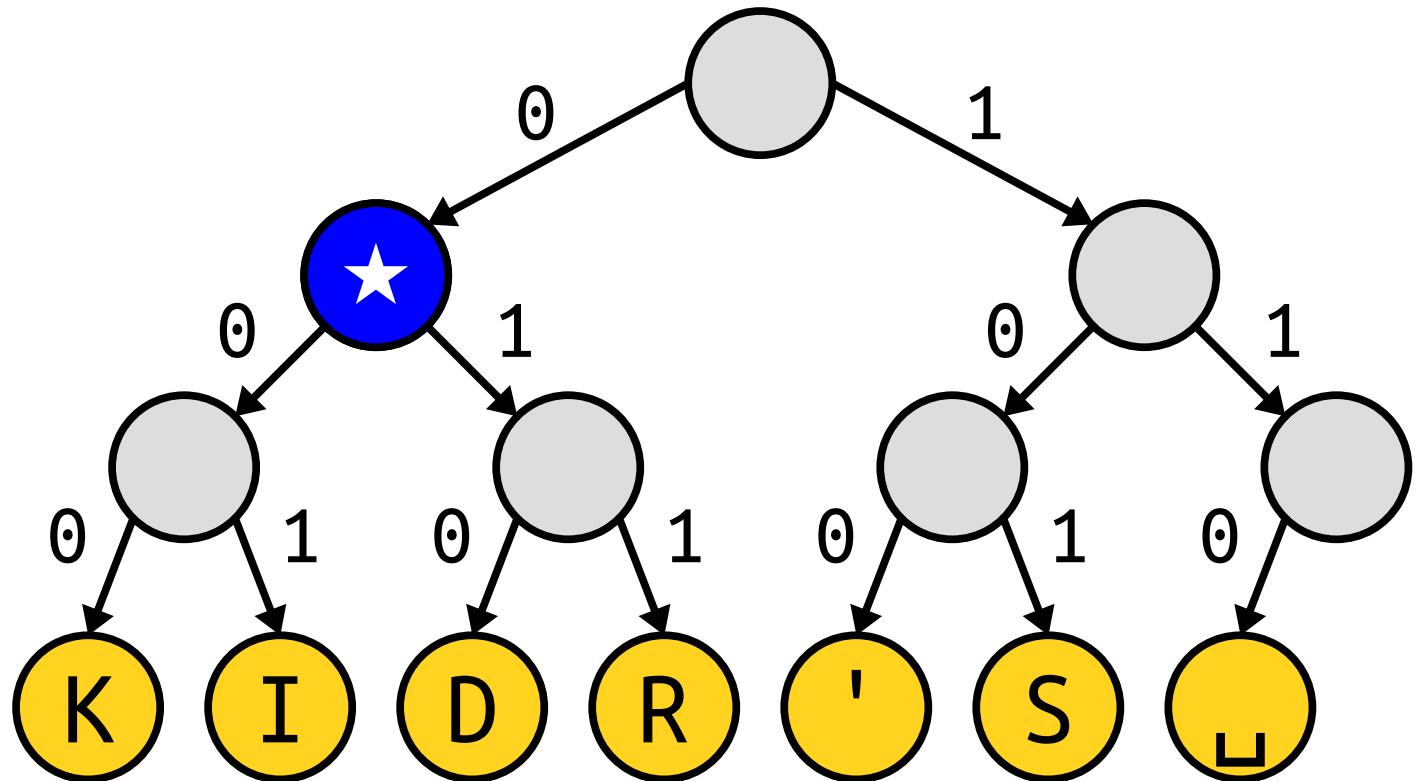
**S K 001**

What is the title of this slide?



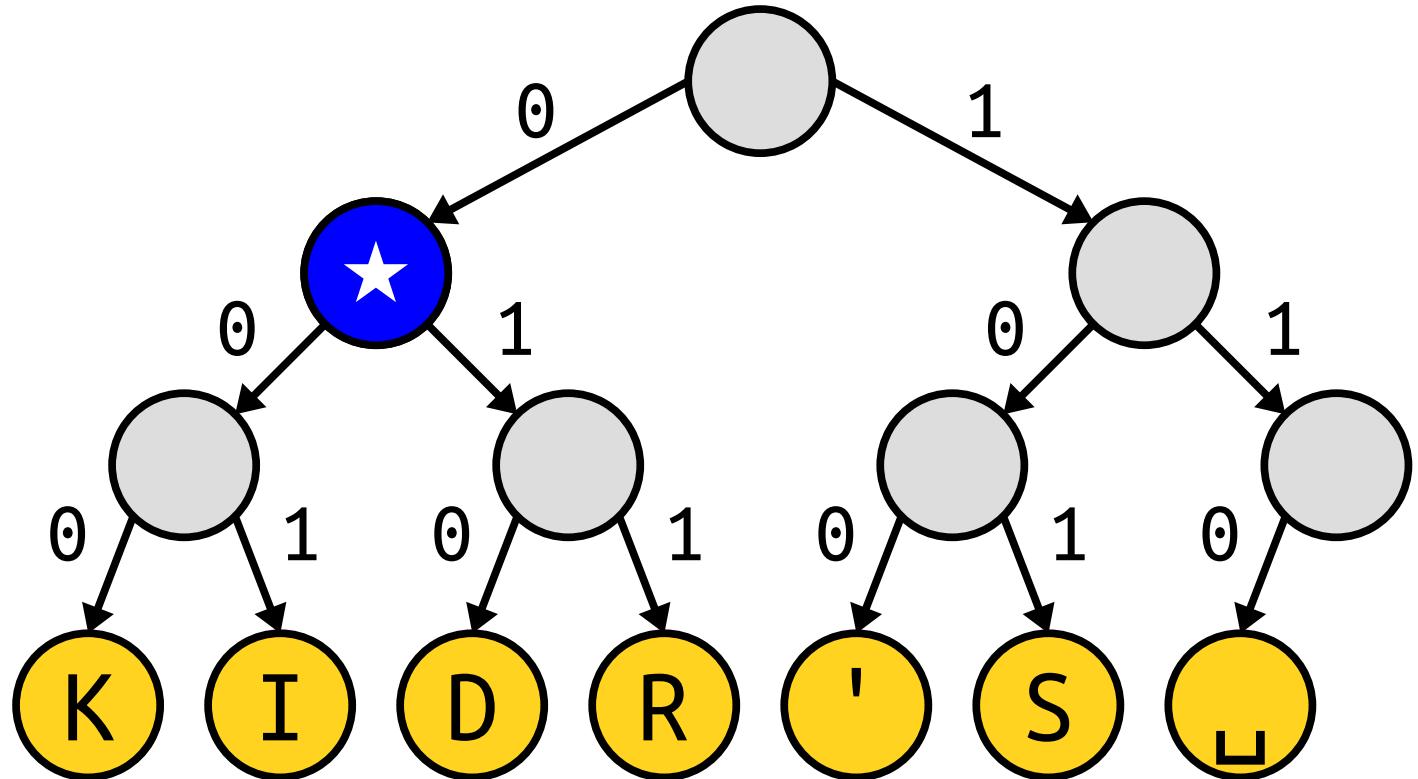
S K 001

What is the title of this slide?



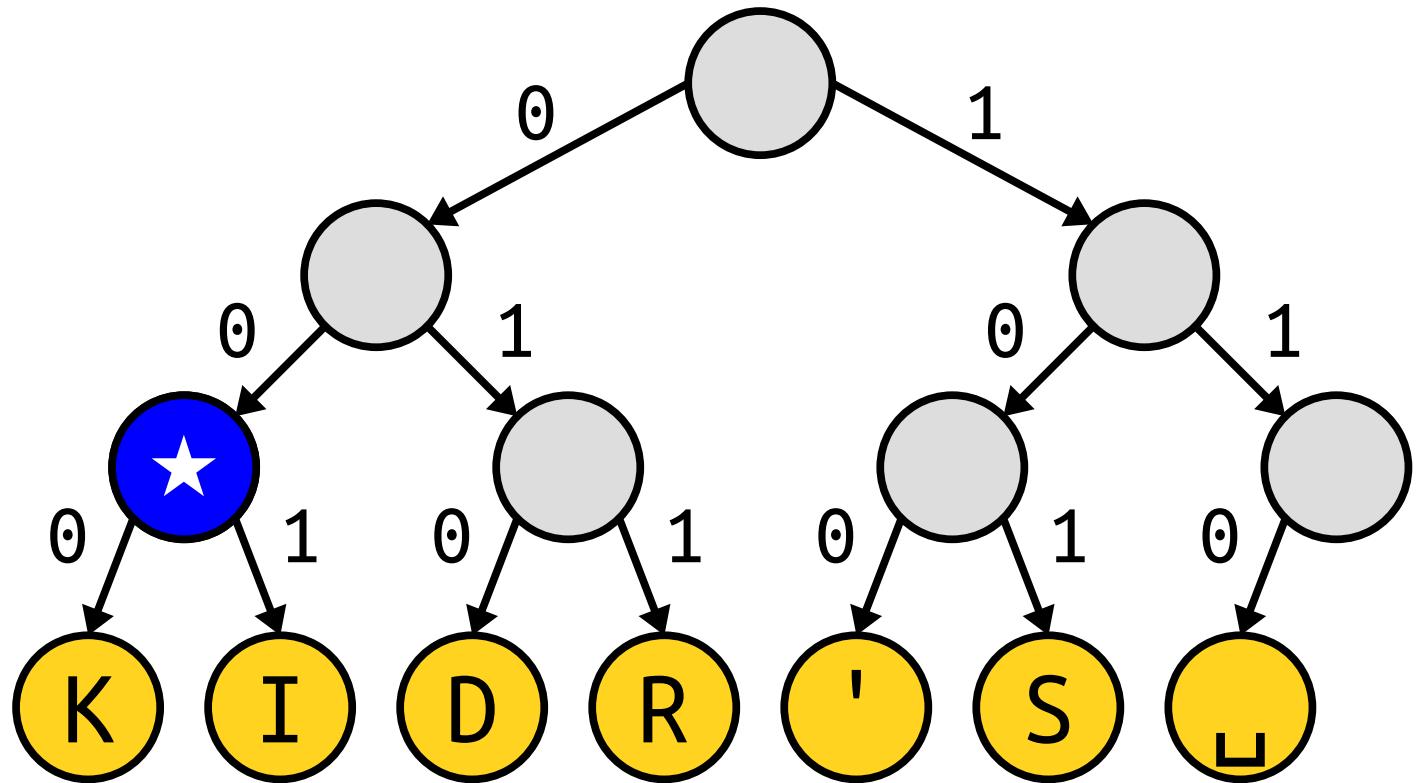
**S K 001**

What is the title of this slide?



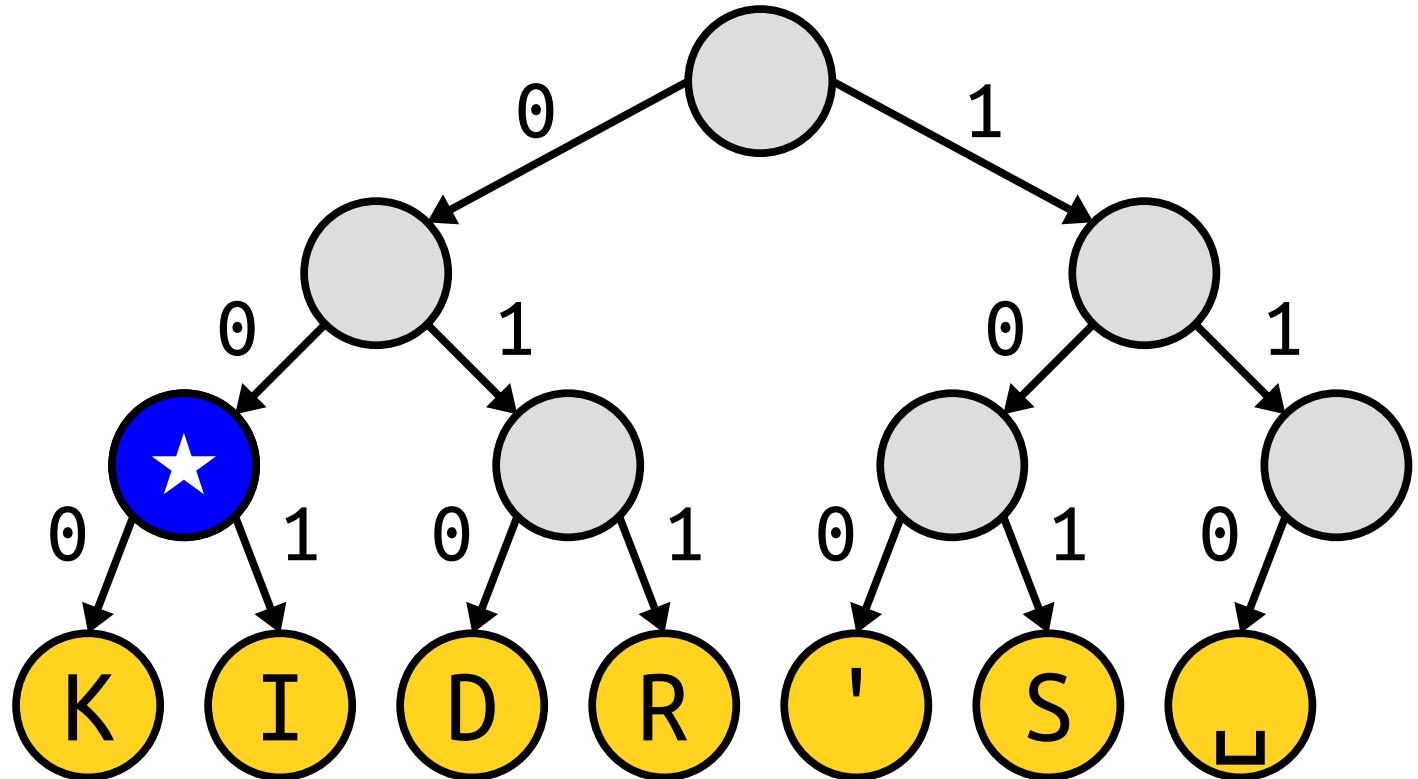
S K 001

What is the title of this slide?



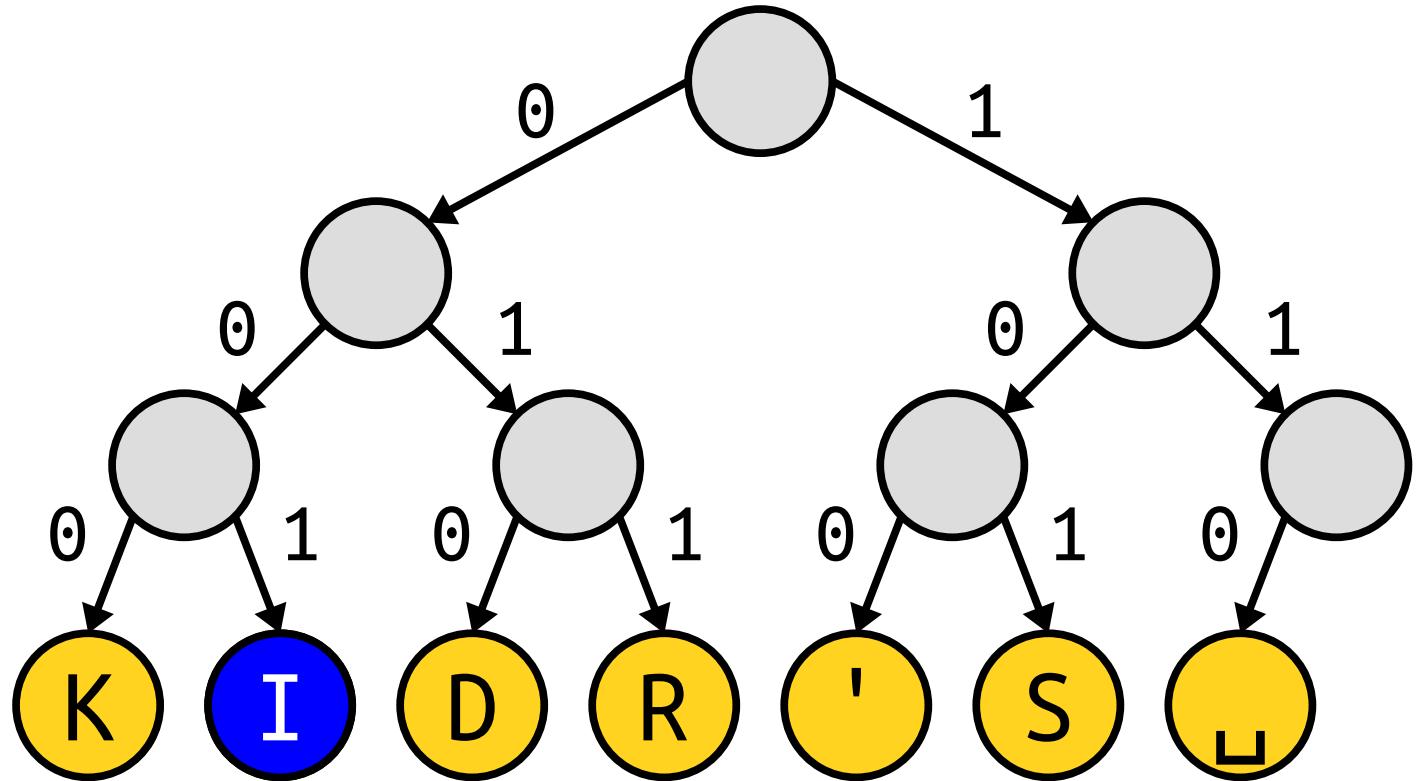
**S K 001**

What is the title of this slide?

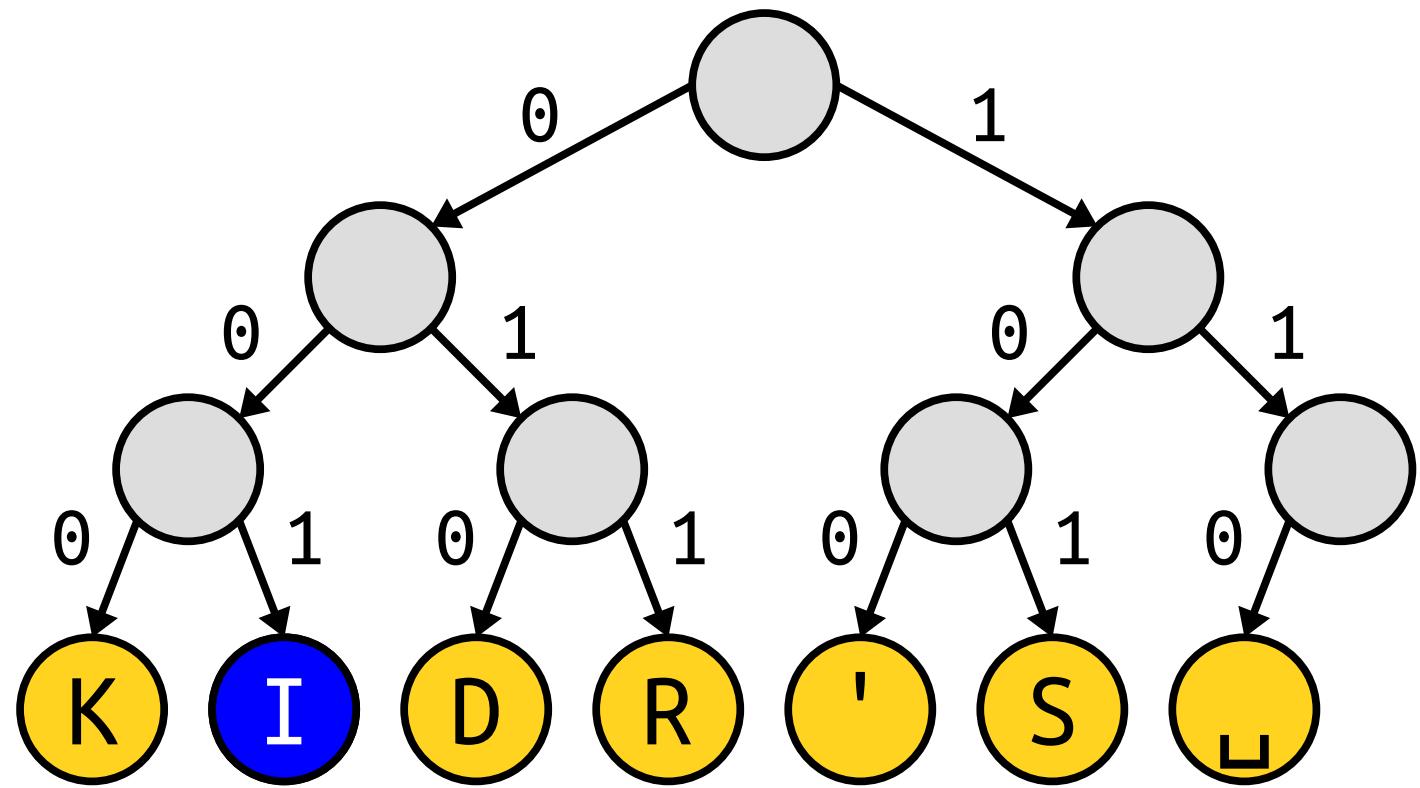


**S K 001**

What is the title of this slide?

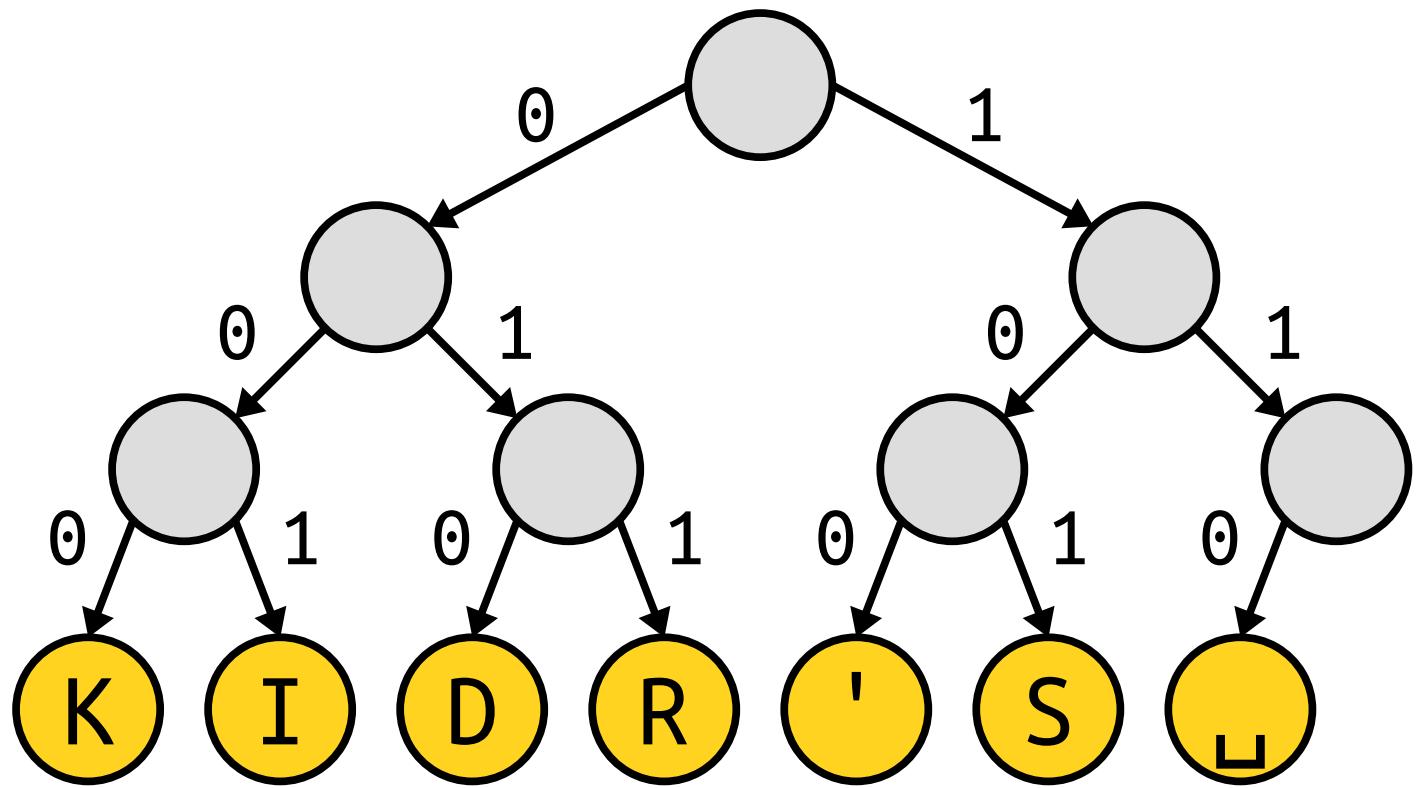


S K I



What is the title of this slide?

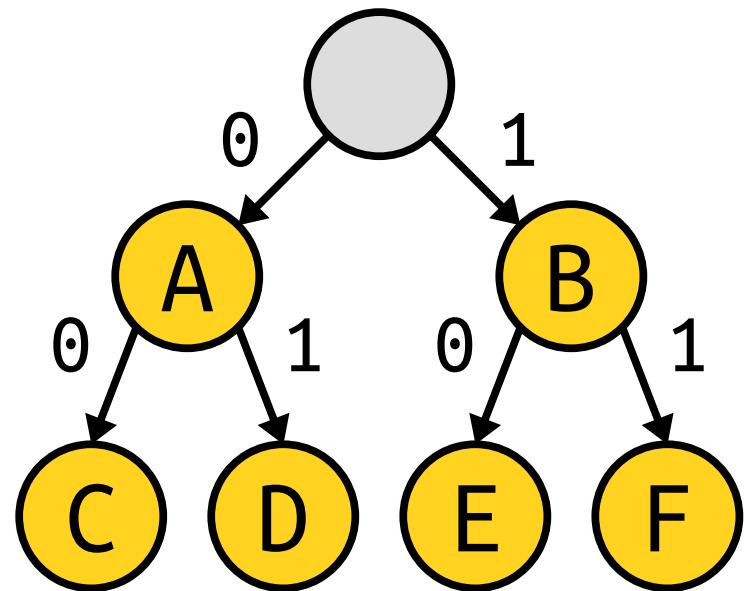
S K I



What is the title of this slide?

# Coding Trees

- Not all binary trees will work as coding trees.
- Why is the one to the right not a valid coding tree?

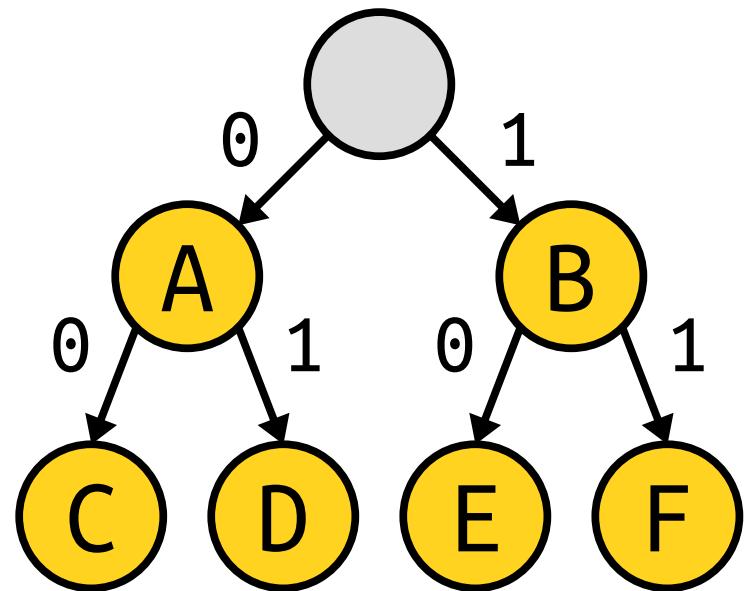


Answer at

<https://pollev.com/cs106bwin23>

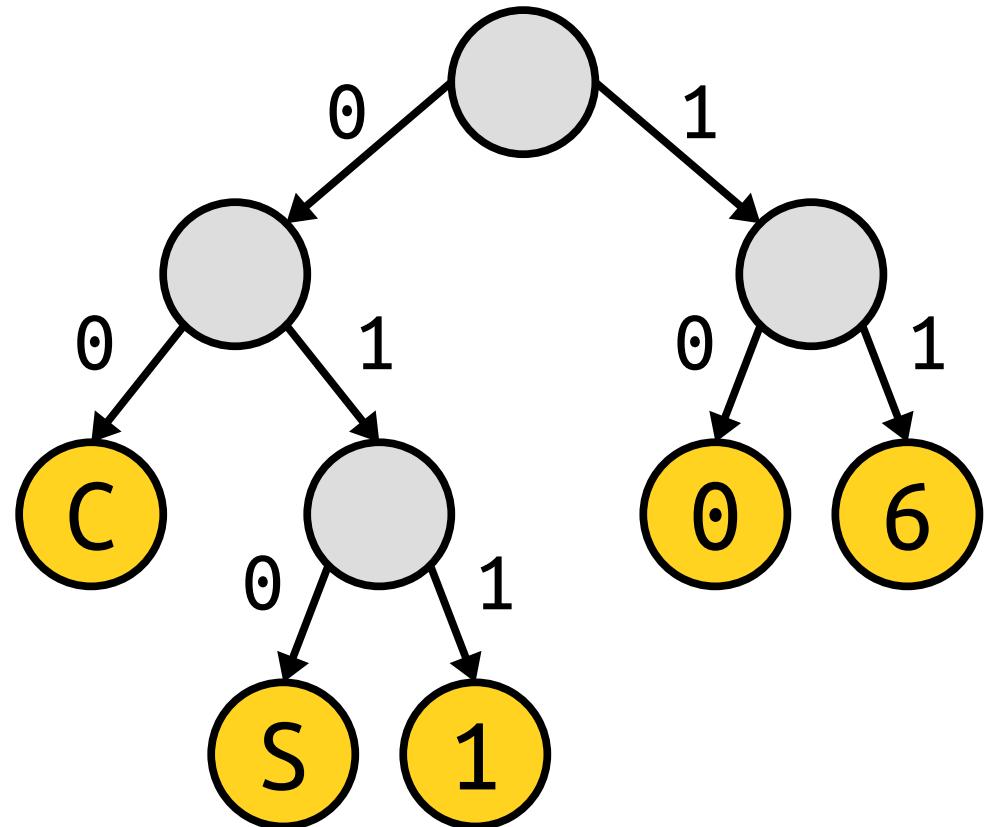
# Coding Trees

- Not all binary trees will work as coding trees.
- Why is the one to the right not a valid coding tree?
- **Answer:** It doesn't give a prefix-free code. The code for A is a prefix for the codes for C and D, and the code for B is a prefix of the codes for E and F.



# Coding Trees

- A coding tree is valid if all the letters are stored at the **leaves**, with internal nodes just doing the routing.
- **Goal:** Find the best coding tree for a string.

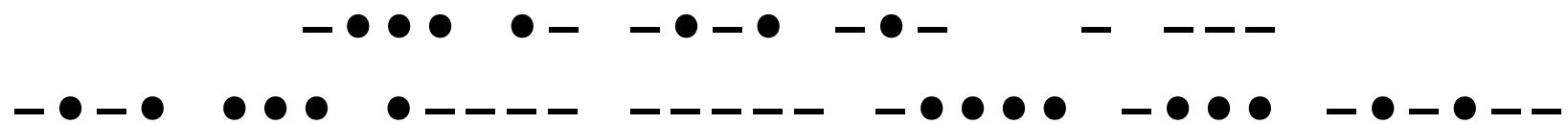


How do we find the best  
coding tree for a piece of text?

Time-Out for Announcements!

# Assignment 8

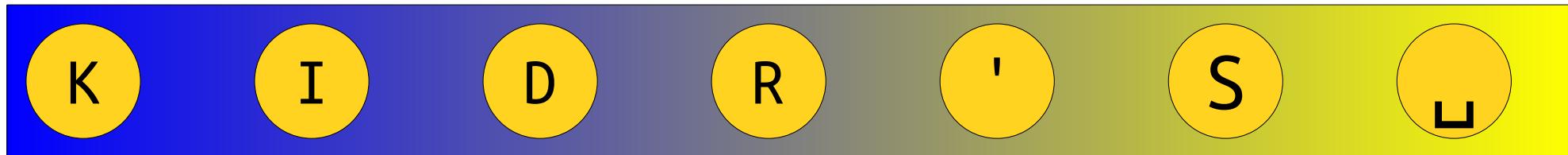
- Assignment 7 was due today at 1:00PM.
  - This is your last chance to use late days this quarter - but don't use them unless you need to.
- Assignment 8 goes out today.
  - Implement the techniques from this lecture!
  - See how much space-saving is available!
  - **No late submissions** will be accepted without prior approval. Sorry - that's university policy.



How do we find the best  
coding tree for a piece of text?

# Huffman Coding

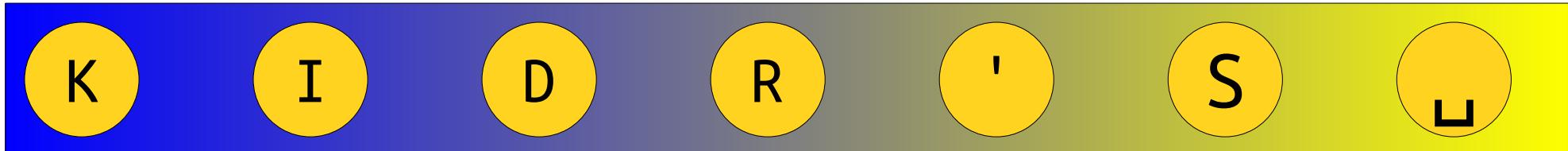
4 3 2 1 1 1 1



*character frequency*

<b>K</b>	<b>4</b>
<b>I</b>	<b>3</b>
<b>D</b>	<b>2</b>
<b>R</b>	<b>1</b>
<b>'</b>	<b>1</b>
<b>S</b>	<b>1</b>
<b>L</b>	<b>1</b>

4 3 2 1 1 1 1



Right now, we have all the leaves of the tree. We now need to build the tree around them.

4

3

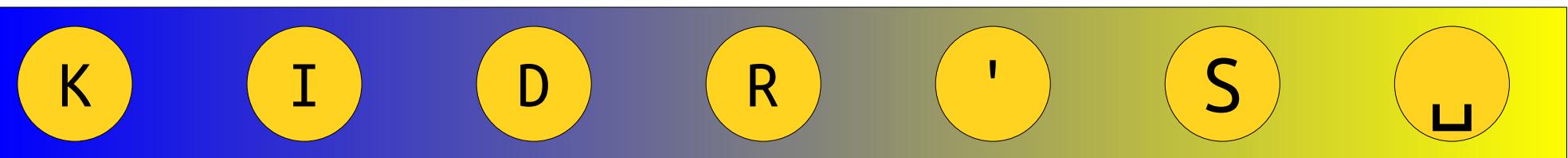
2

1

1

1

1



4

3

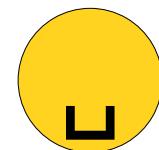
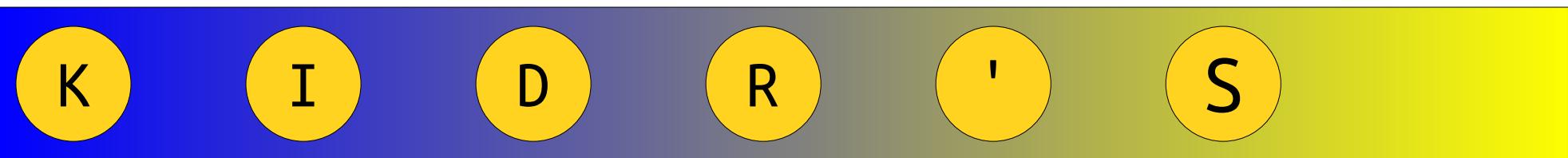
2

1

1

1

1



1

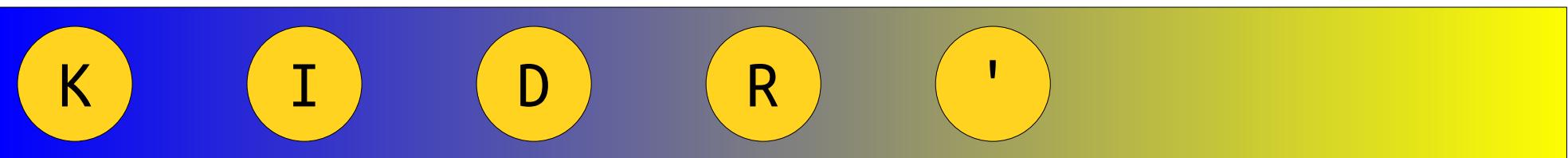
4

3

2

1

1



□ **S**  
1 1

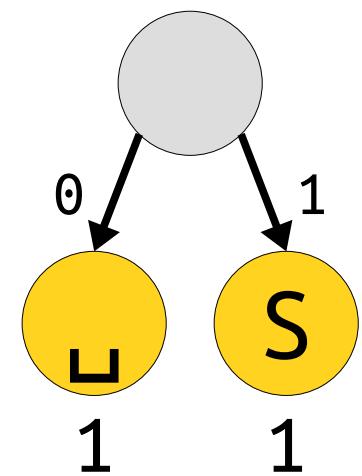
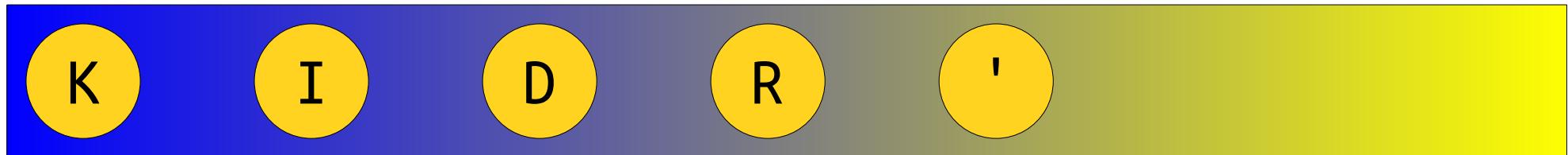
4

3

2

1

1



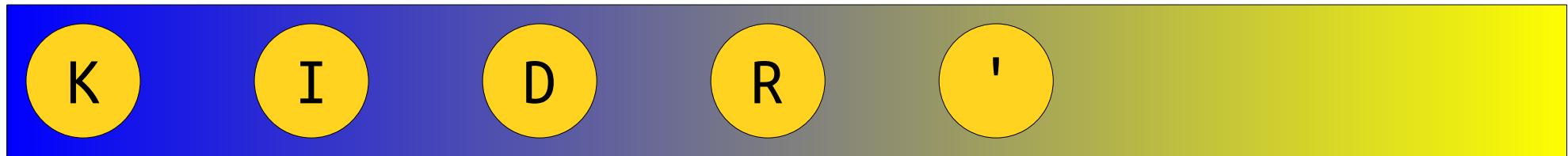
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3

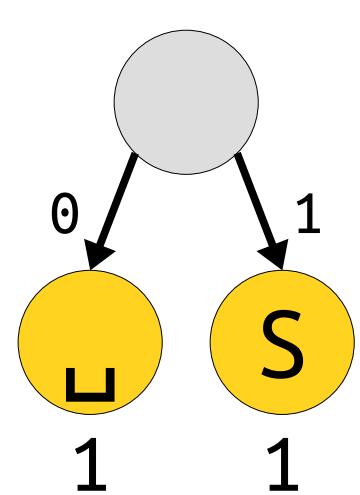
2

1

1



2



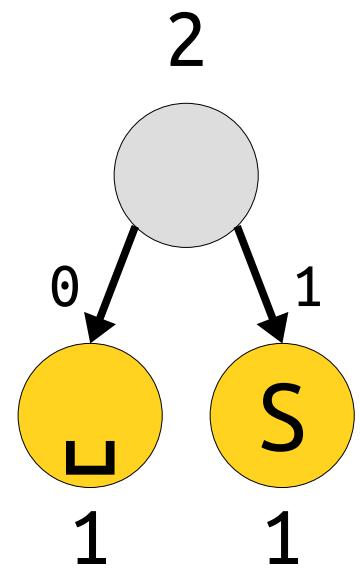
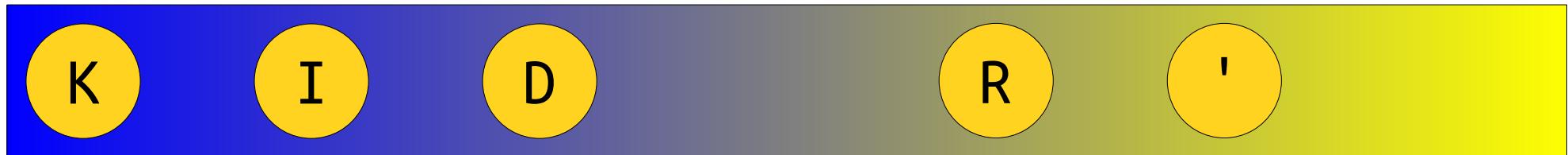
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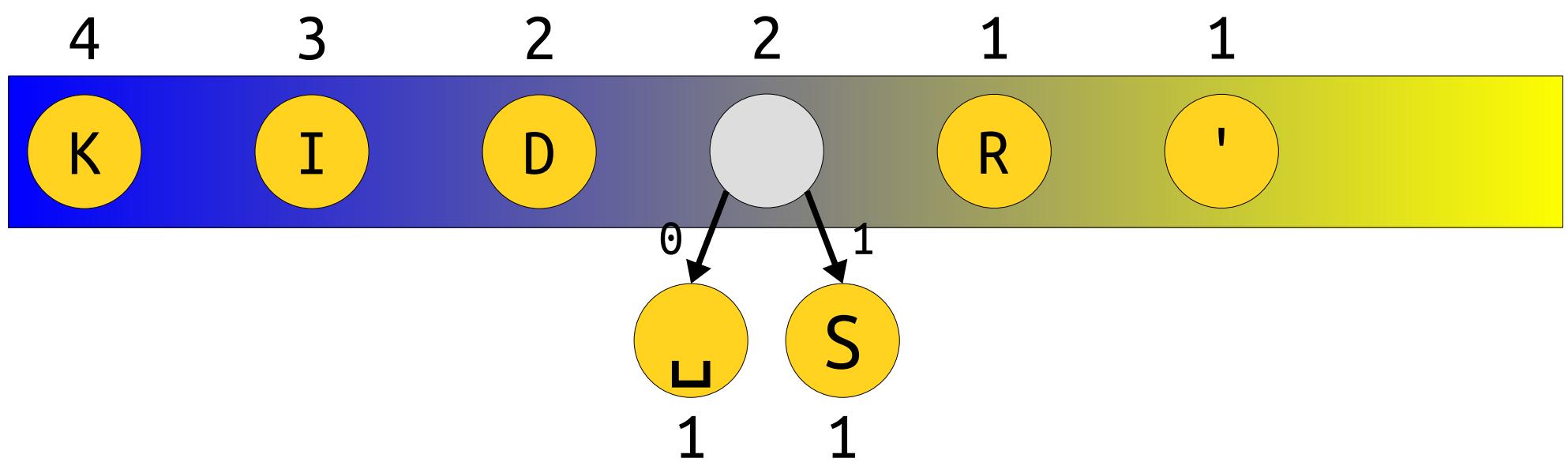
3

2

1

1





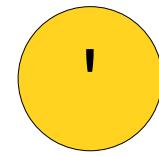
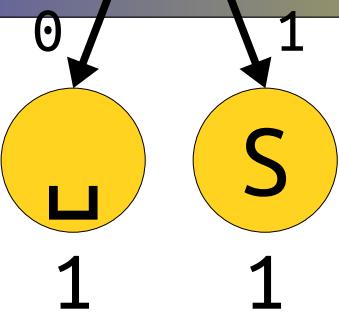
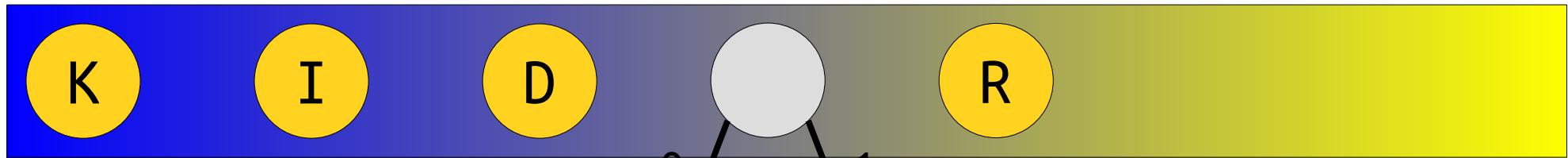
4

3

2

2

1



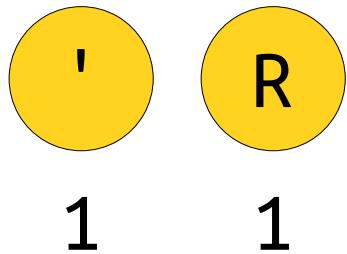
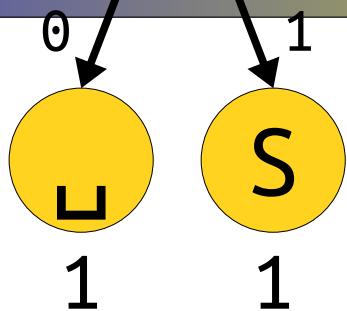
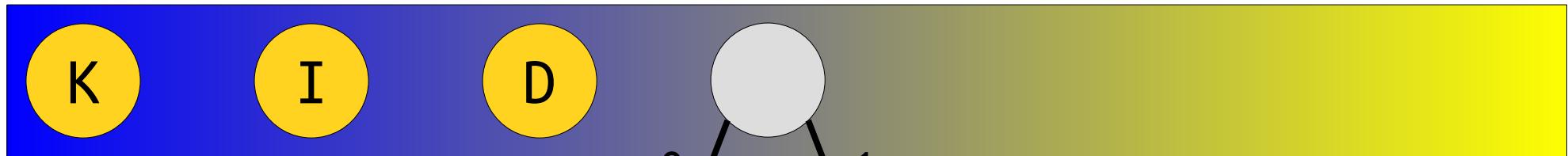
1

4

3

2

2

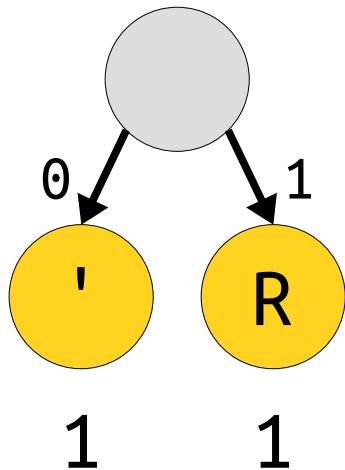
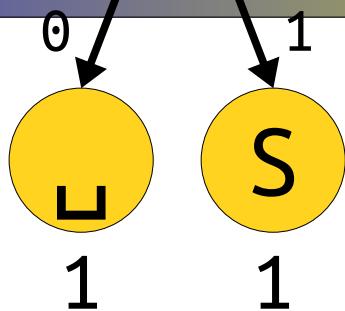
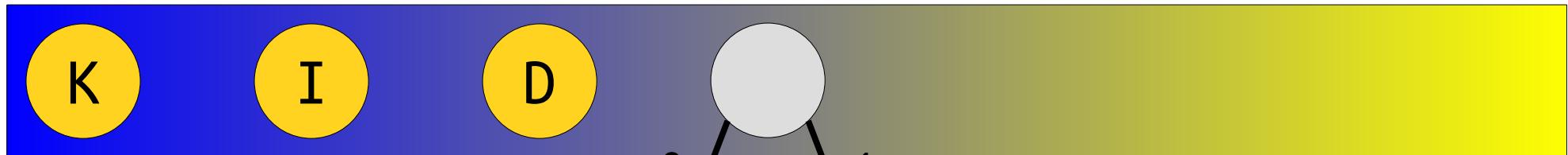


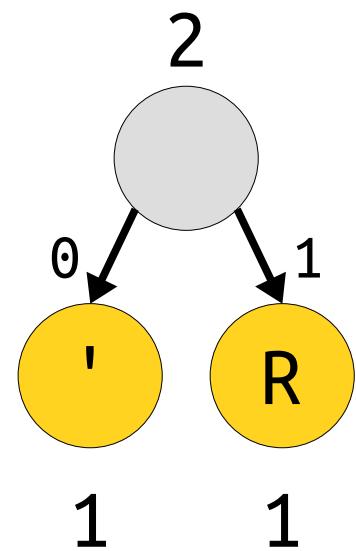
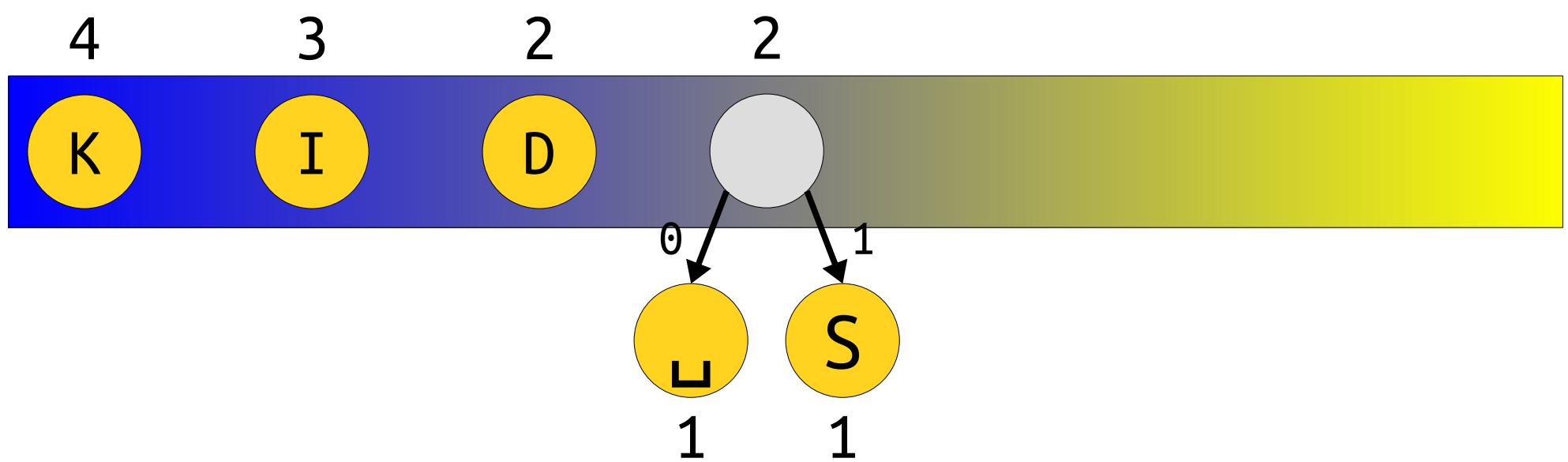
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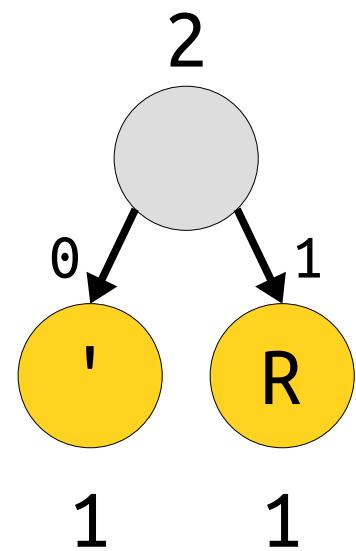
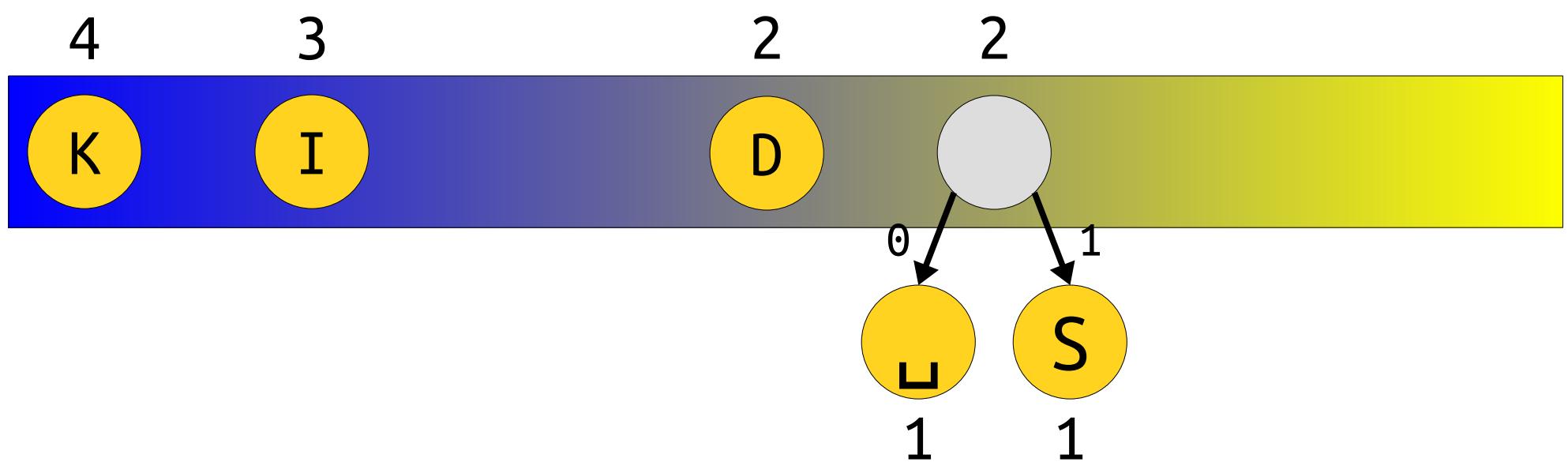
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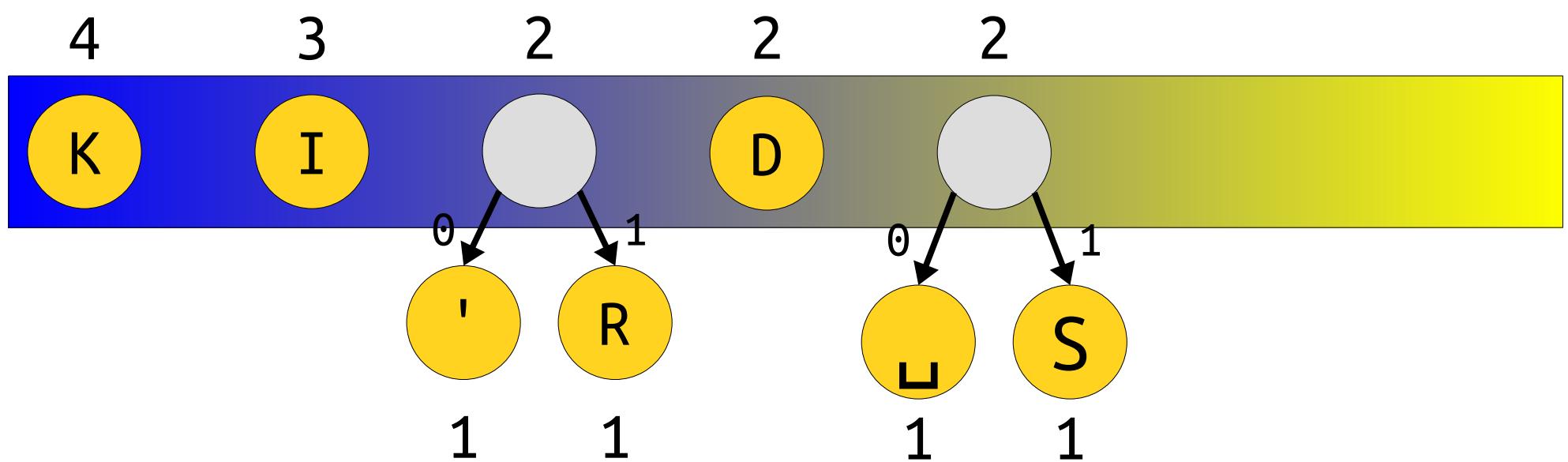
2

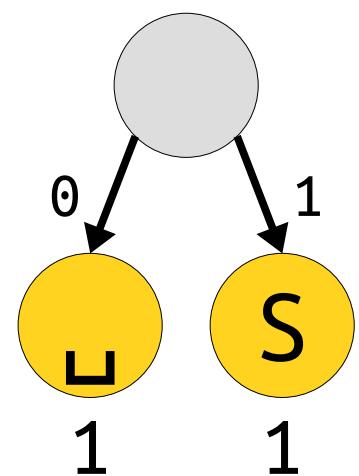
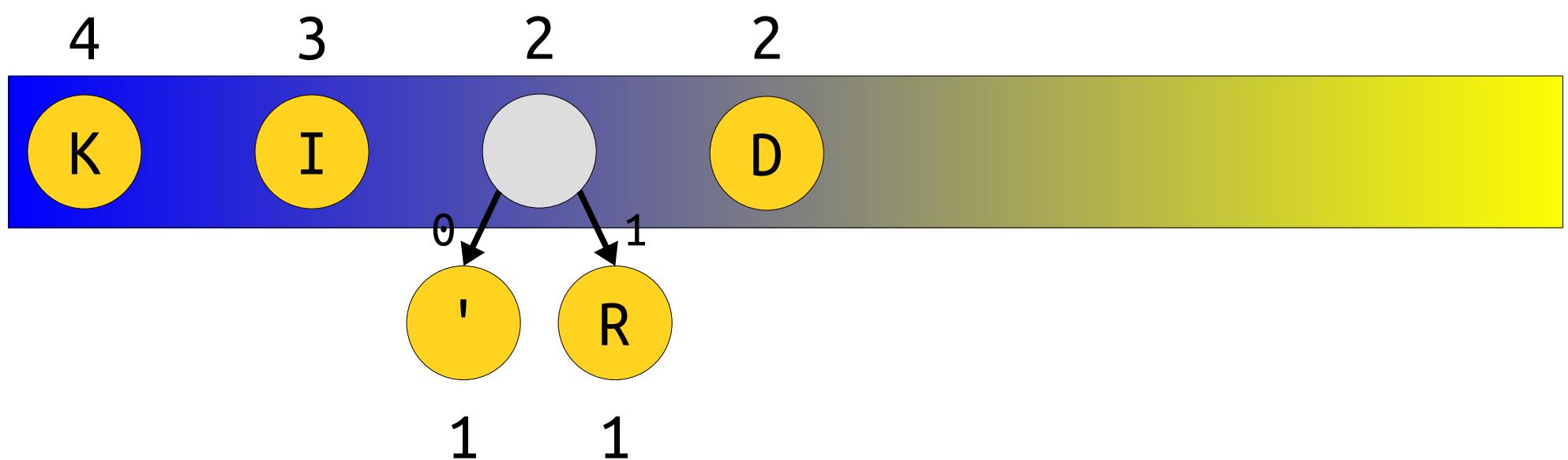
2







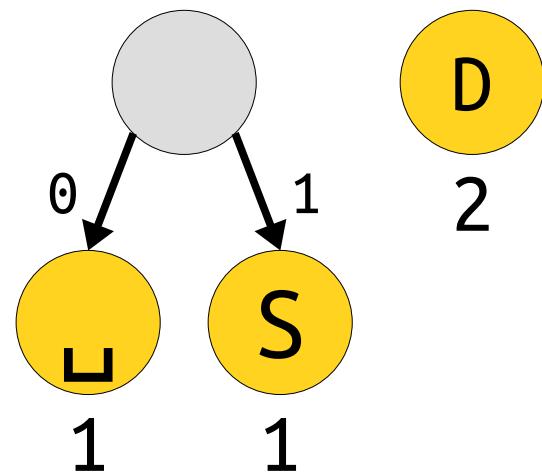
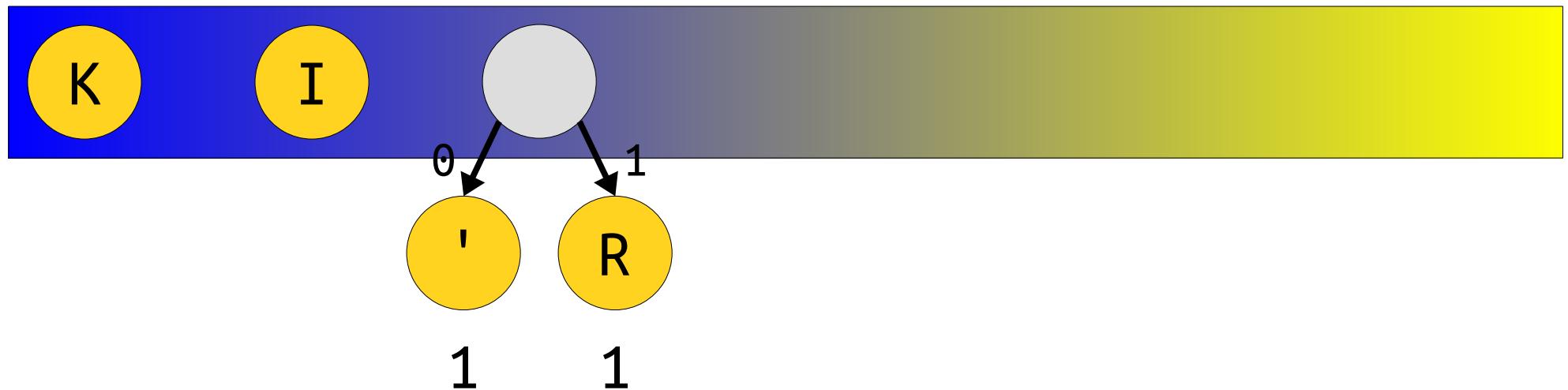


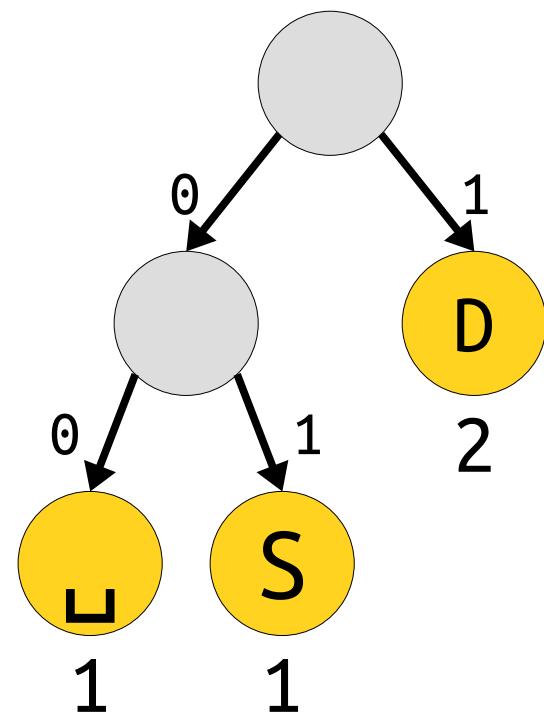
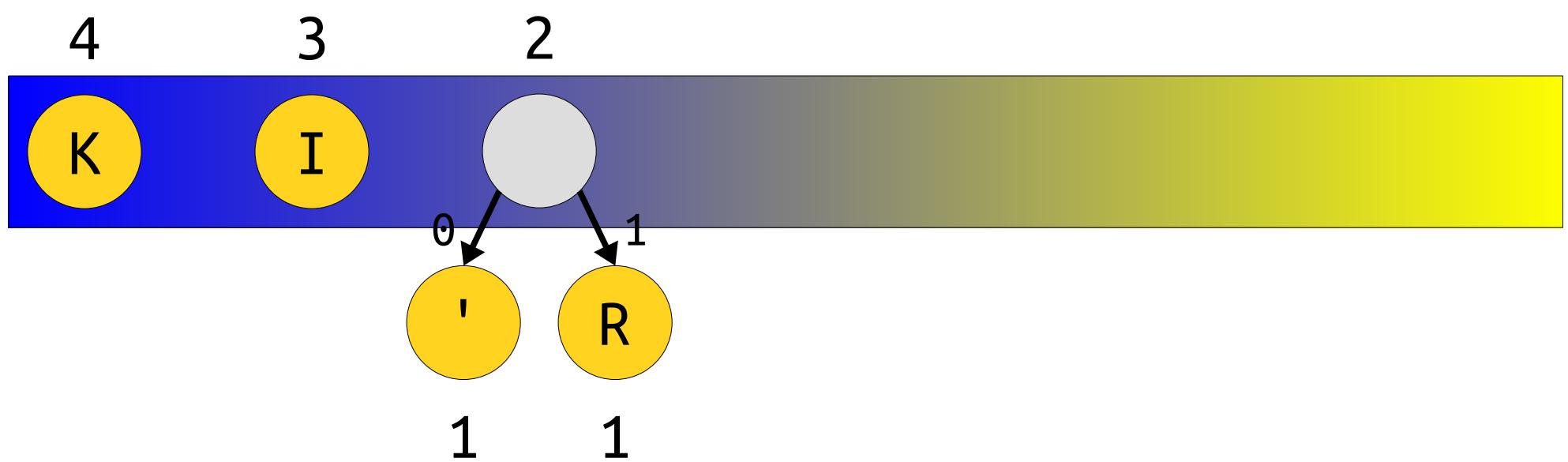


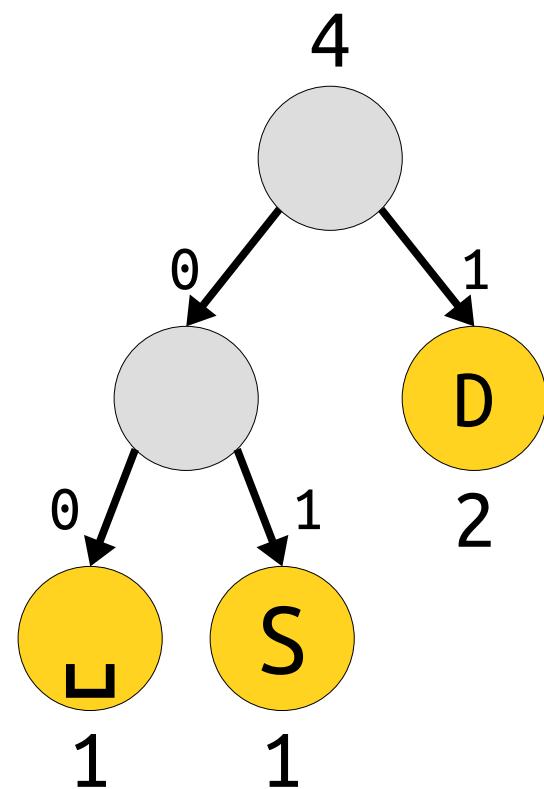
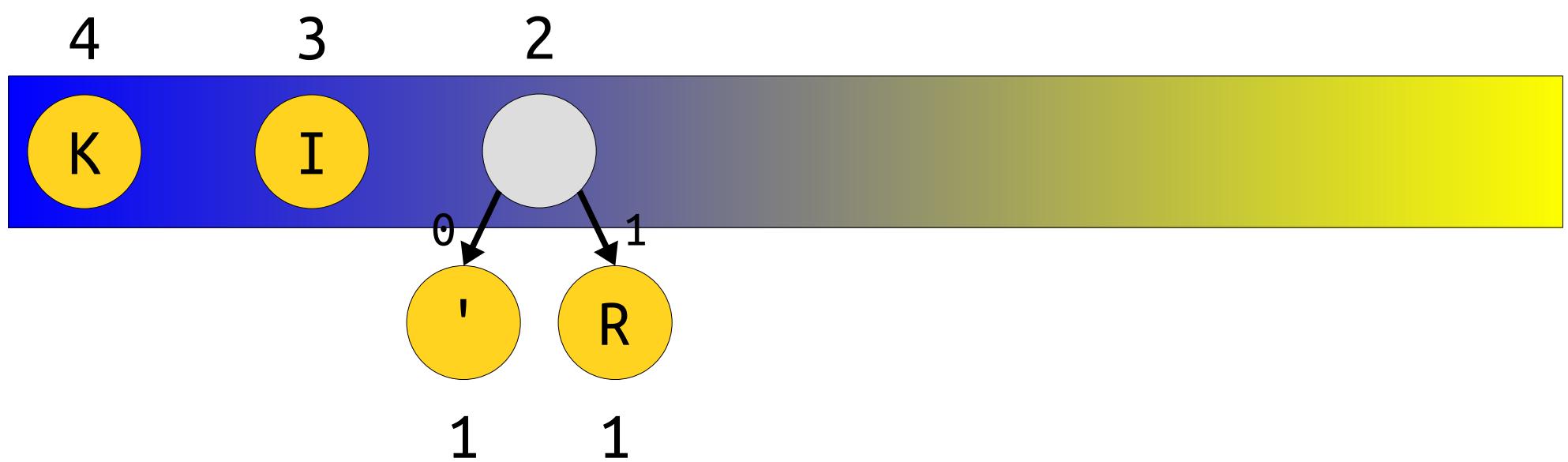
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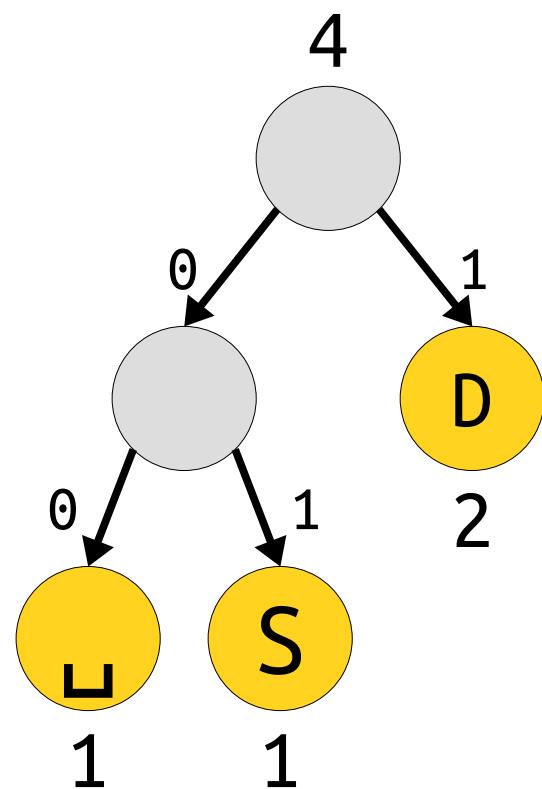
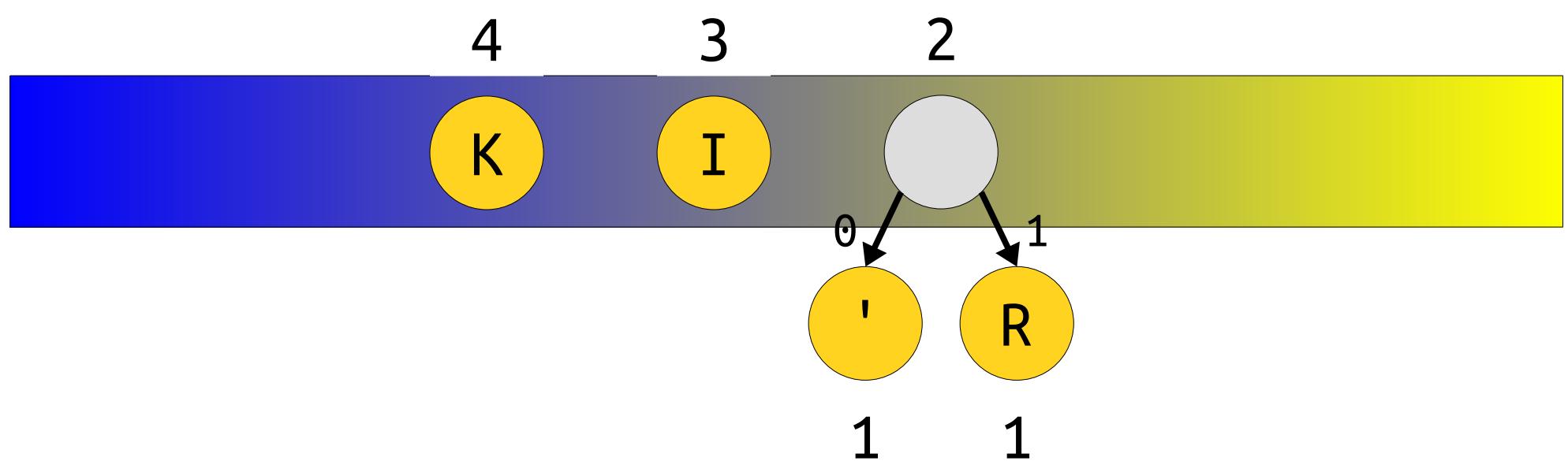
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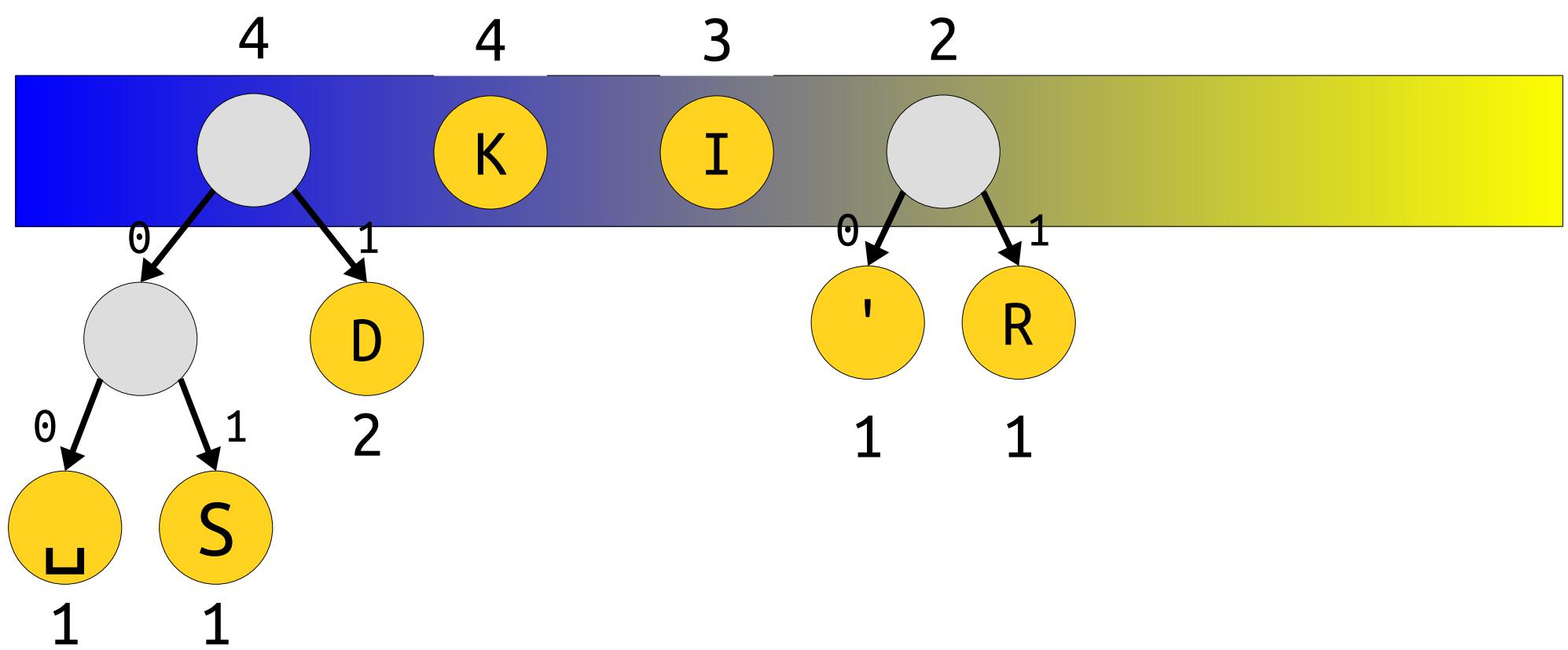
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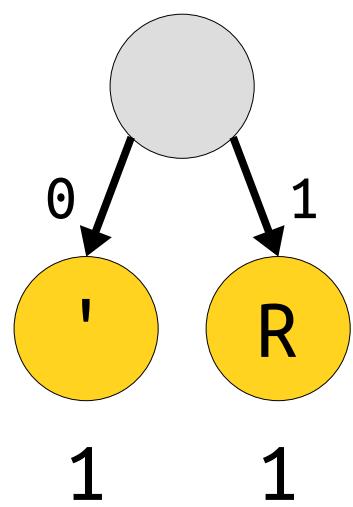
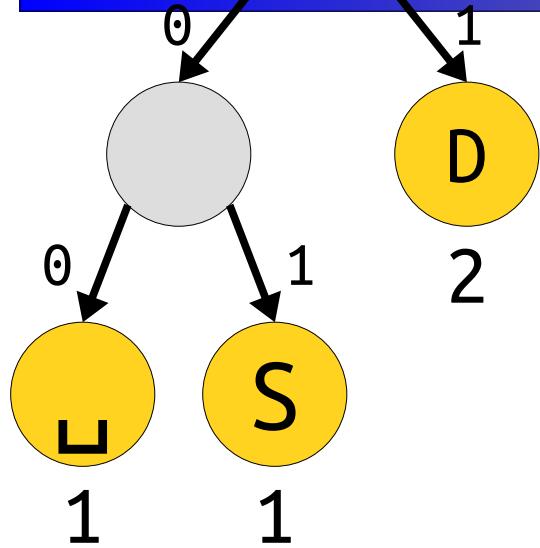
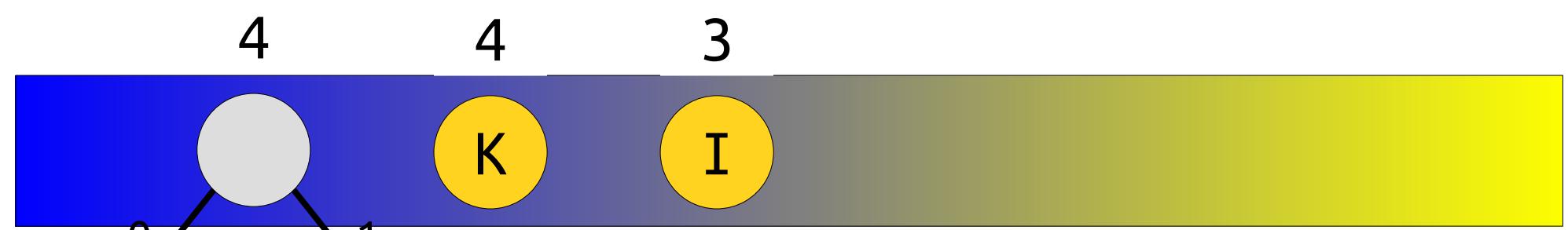


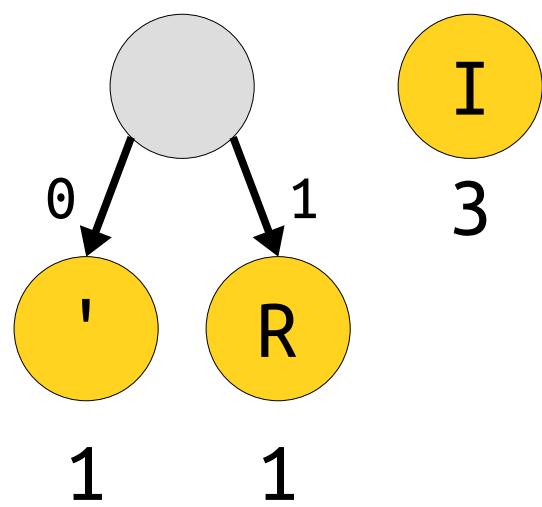
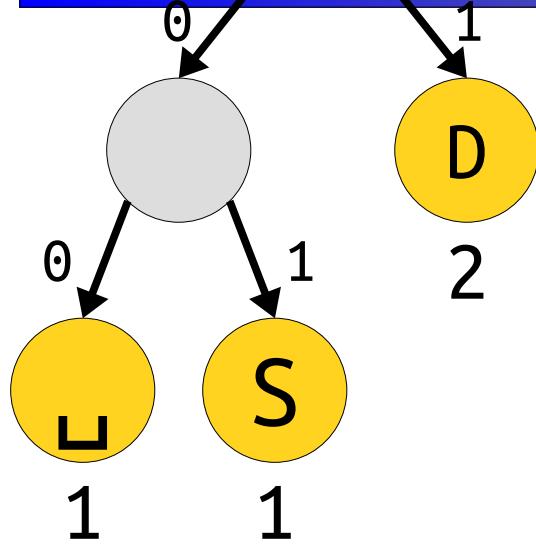


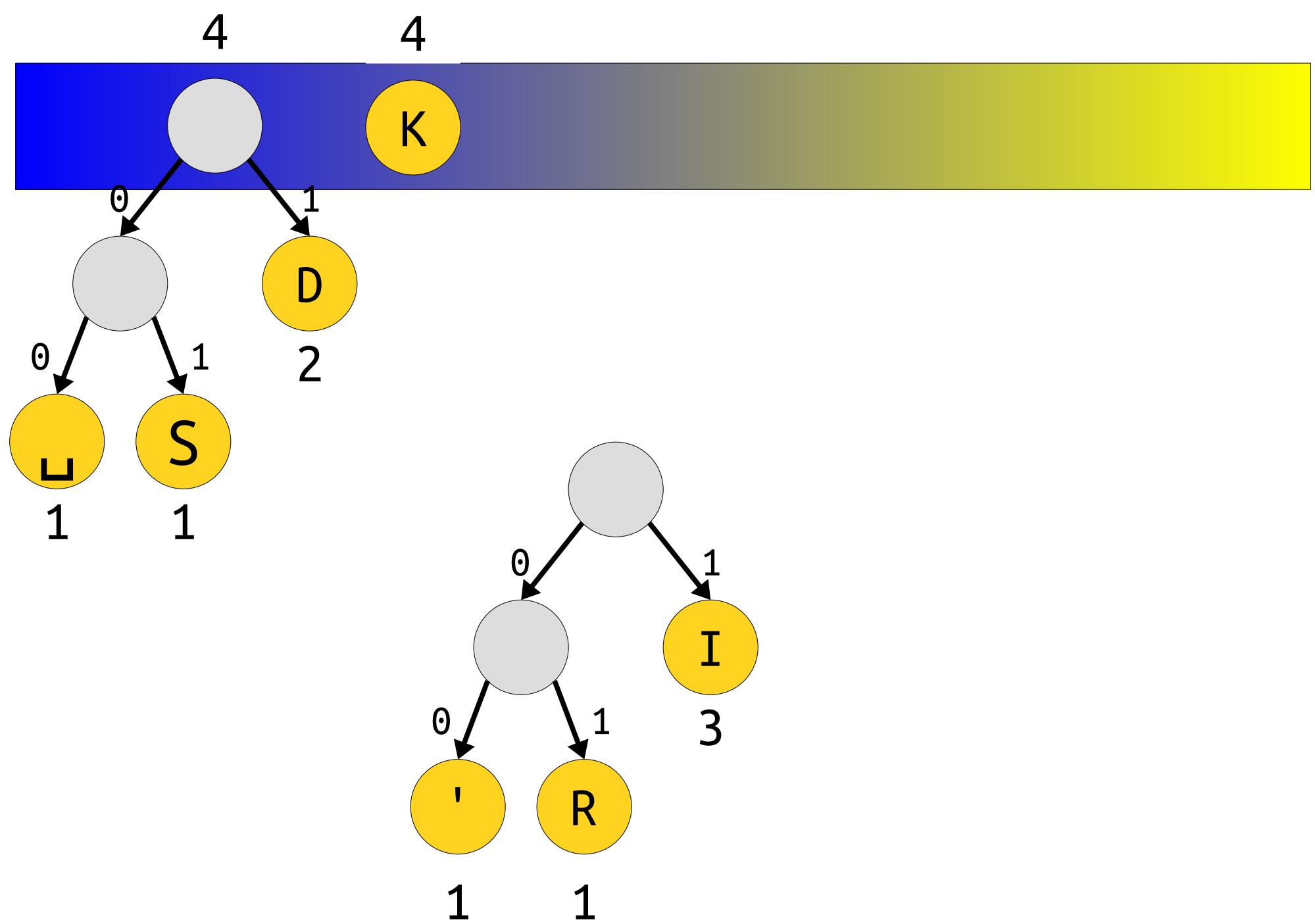


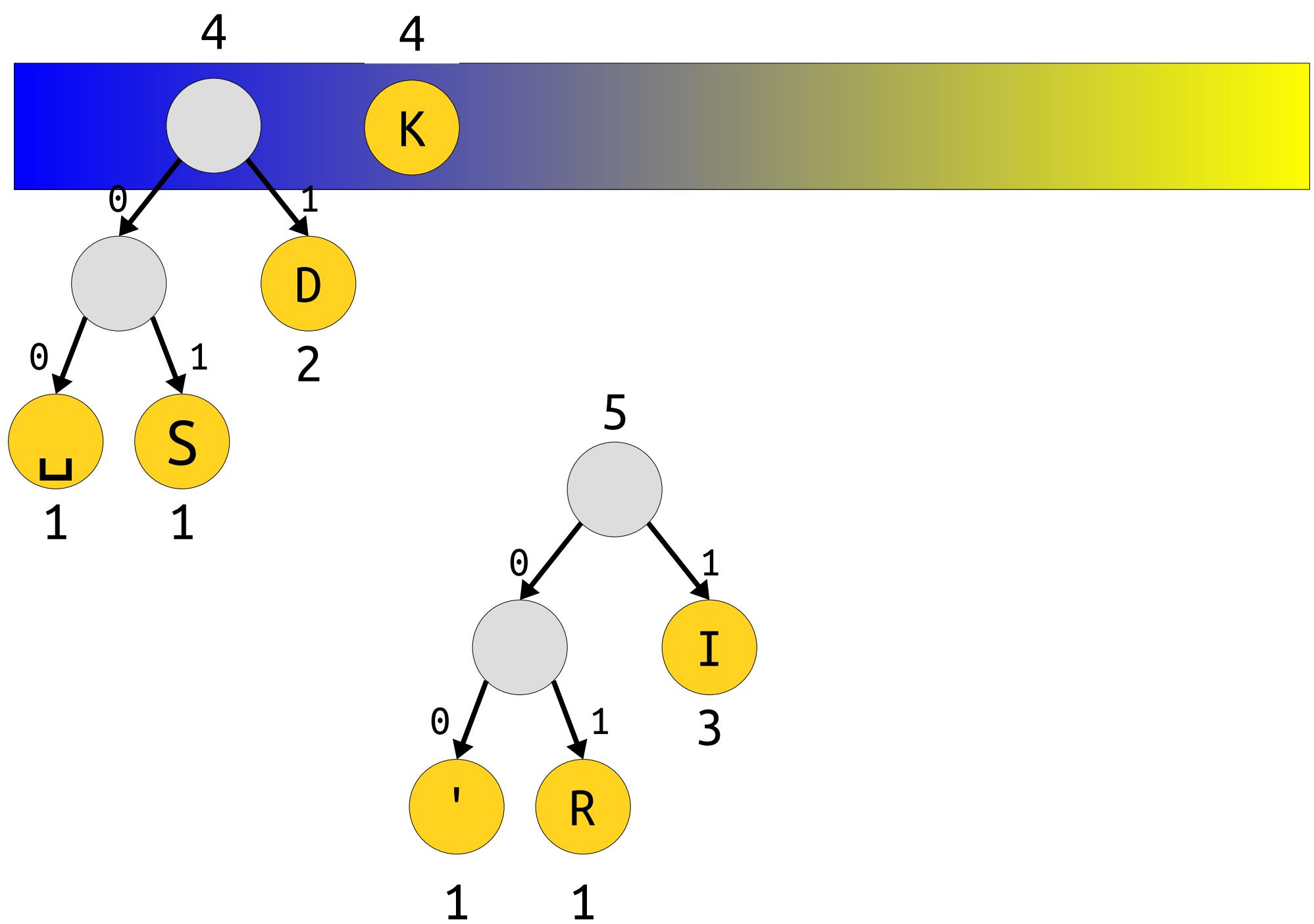


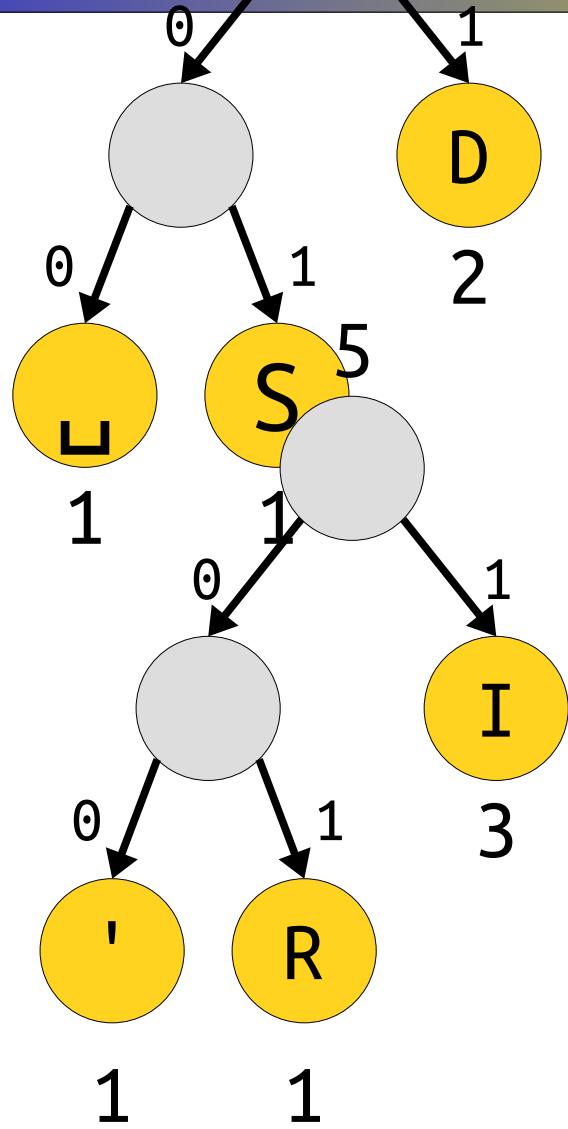
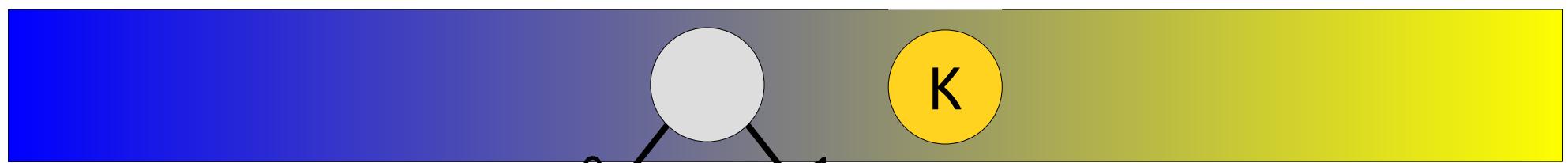








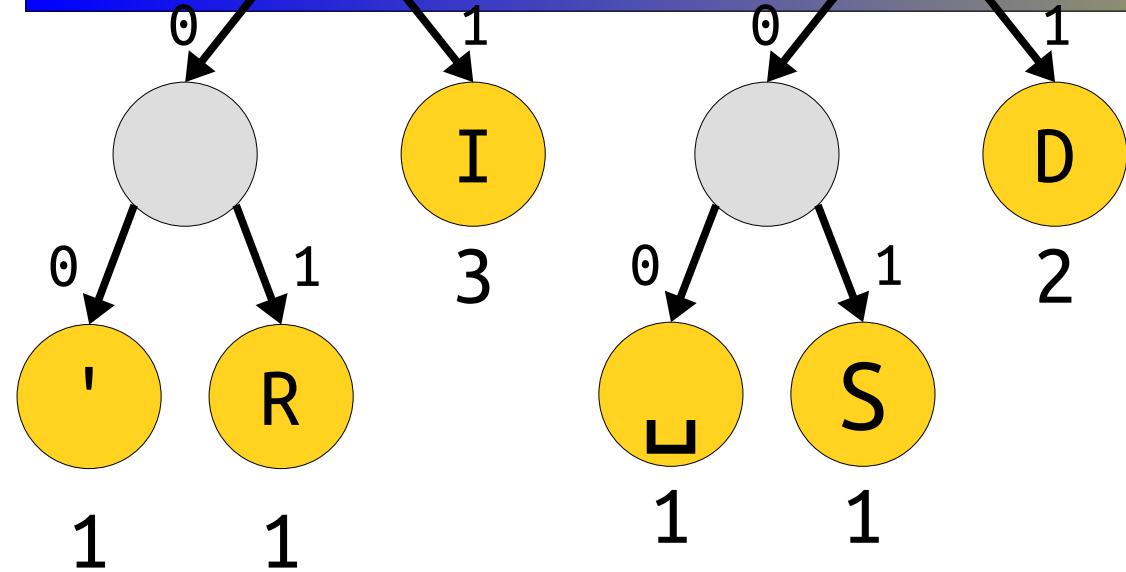
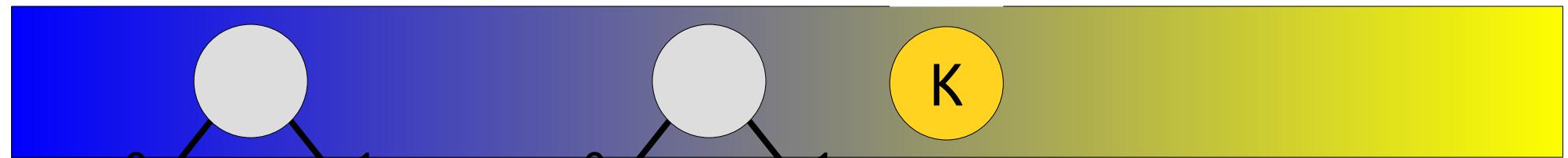




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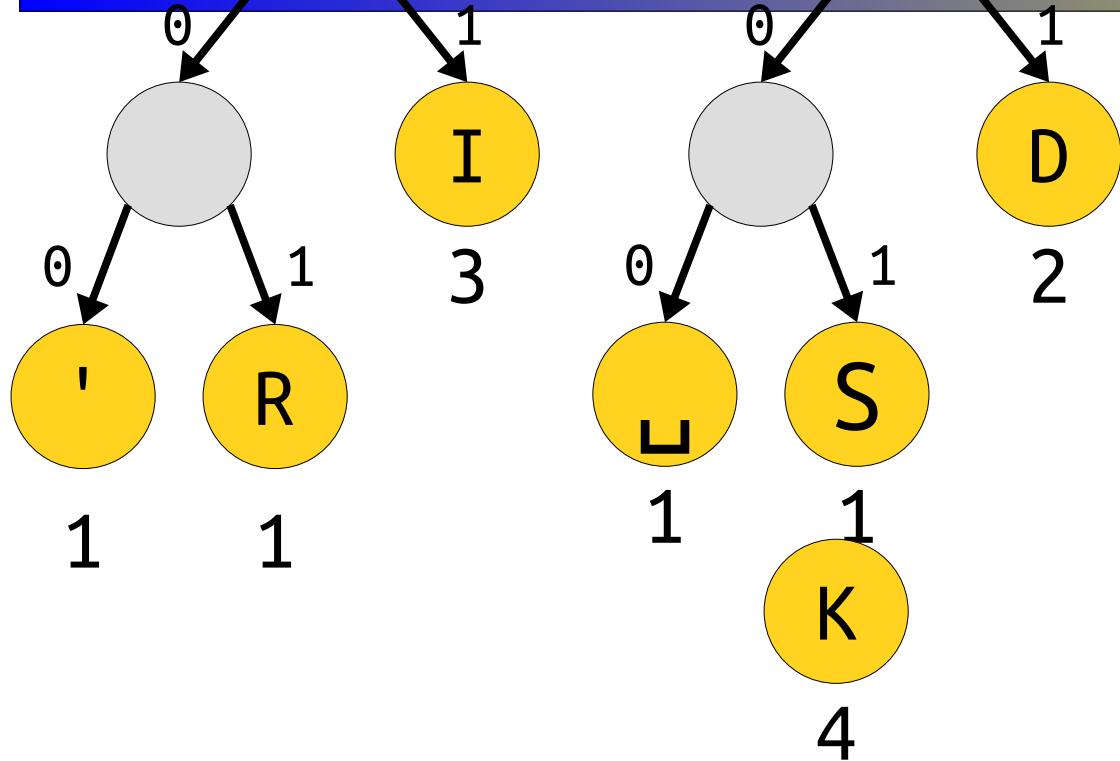
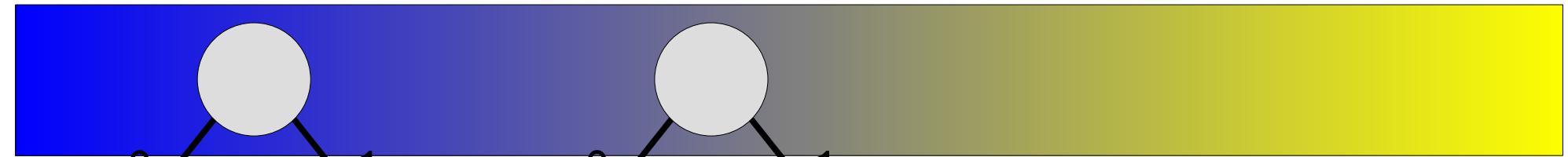
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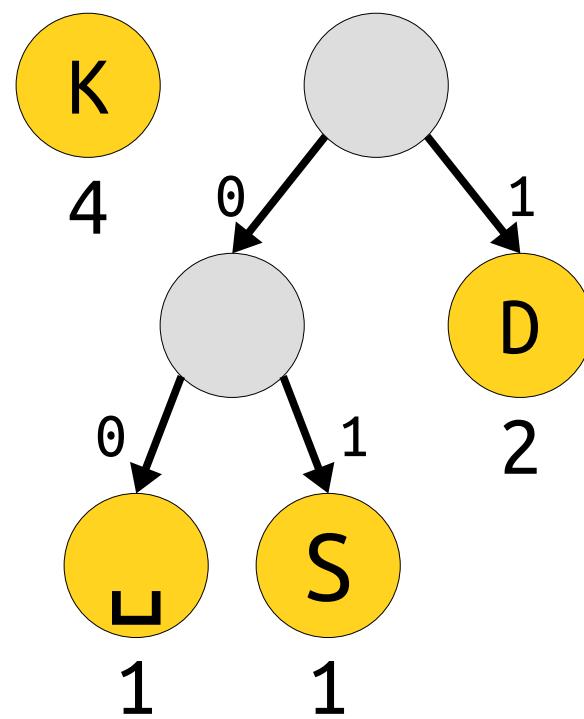
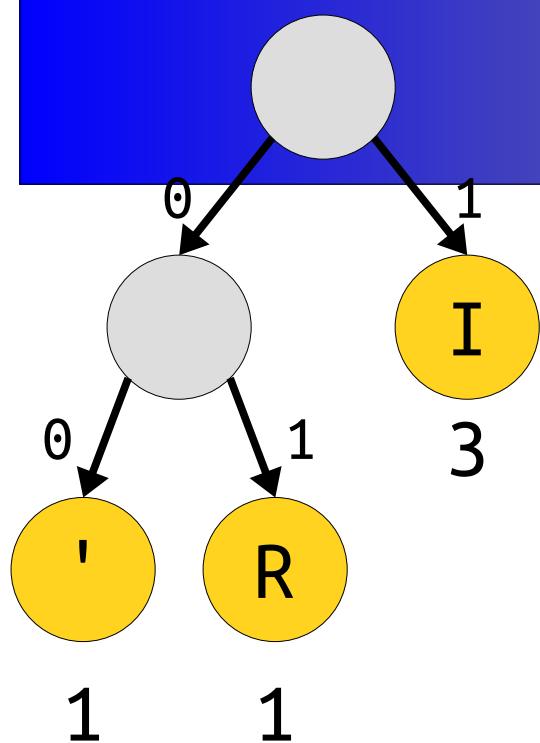


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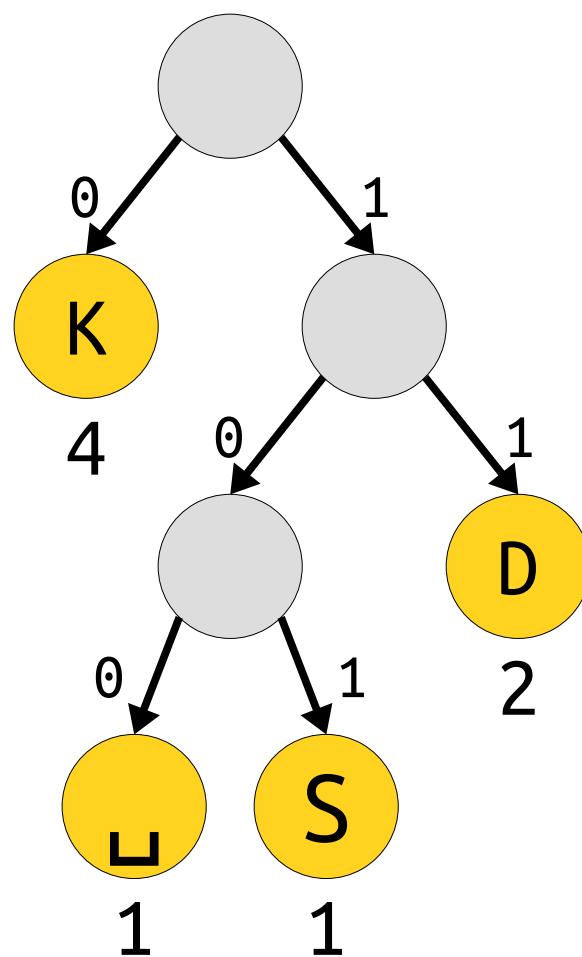
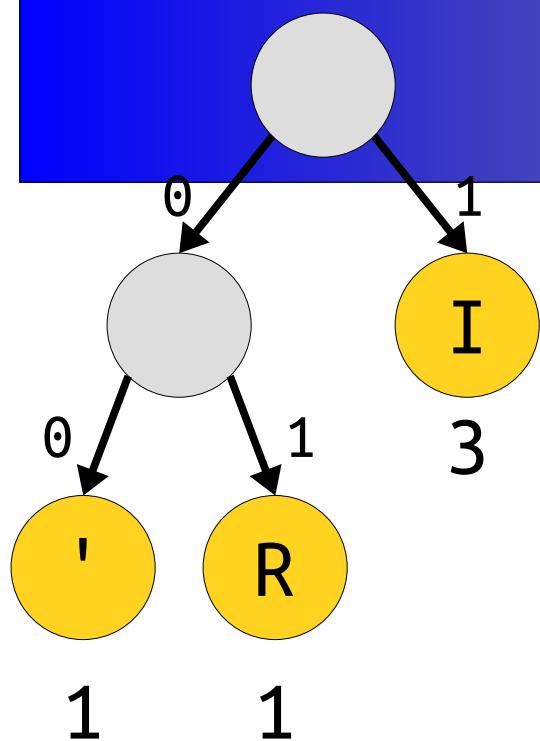
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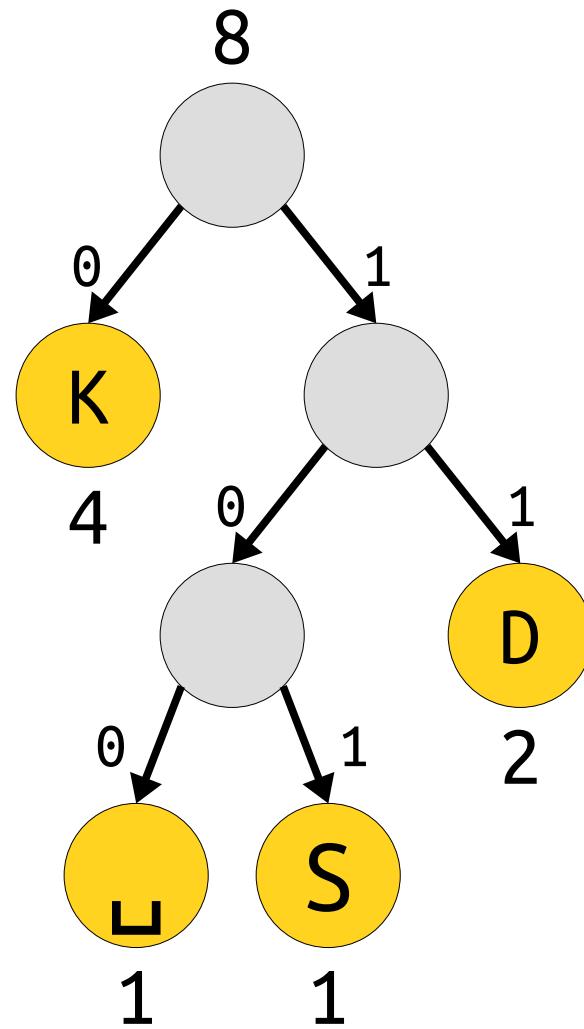
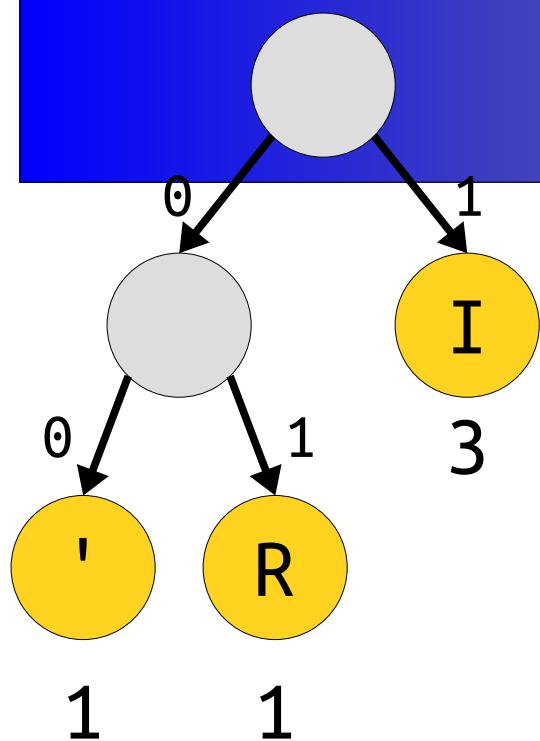
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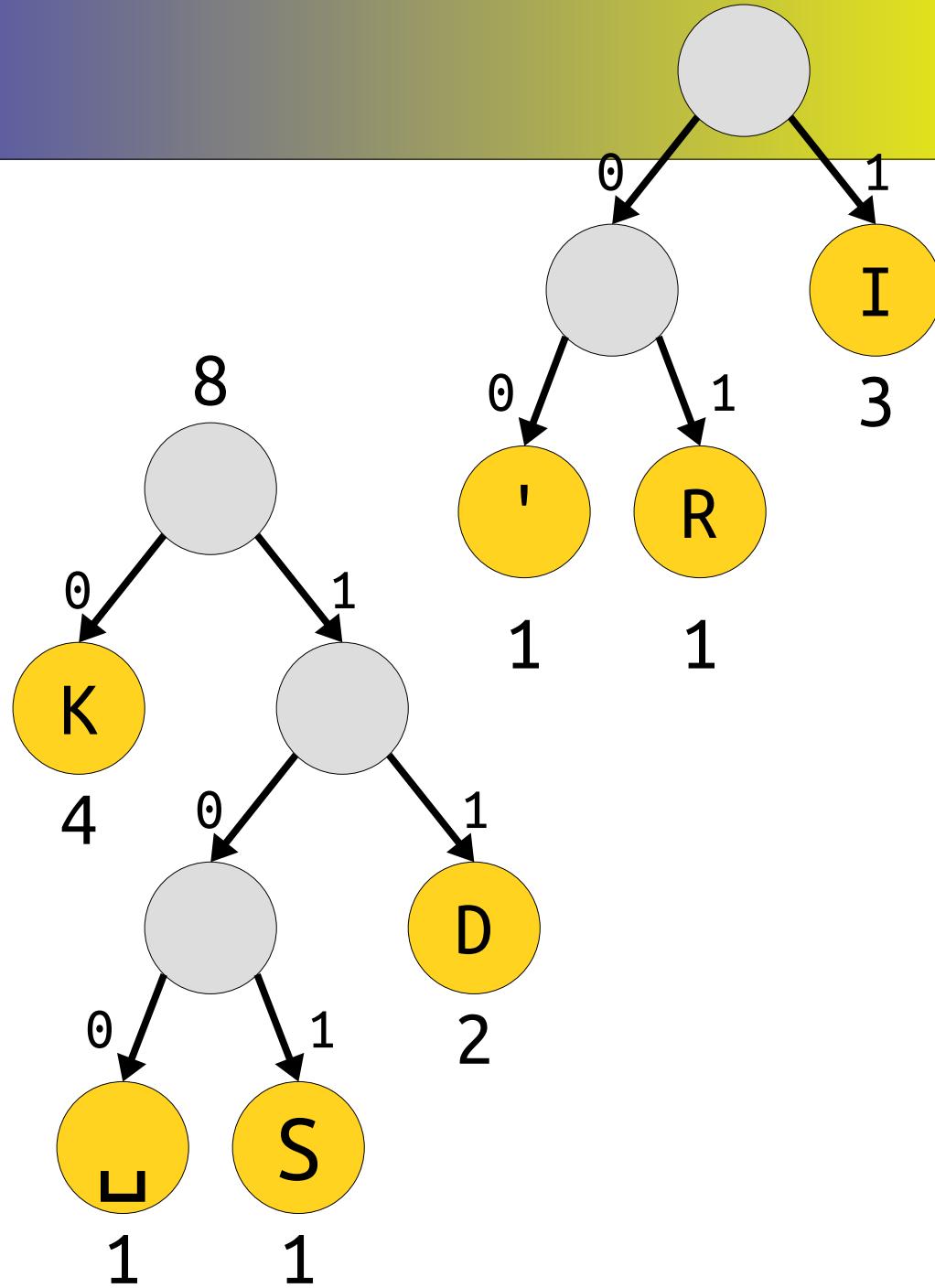
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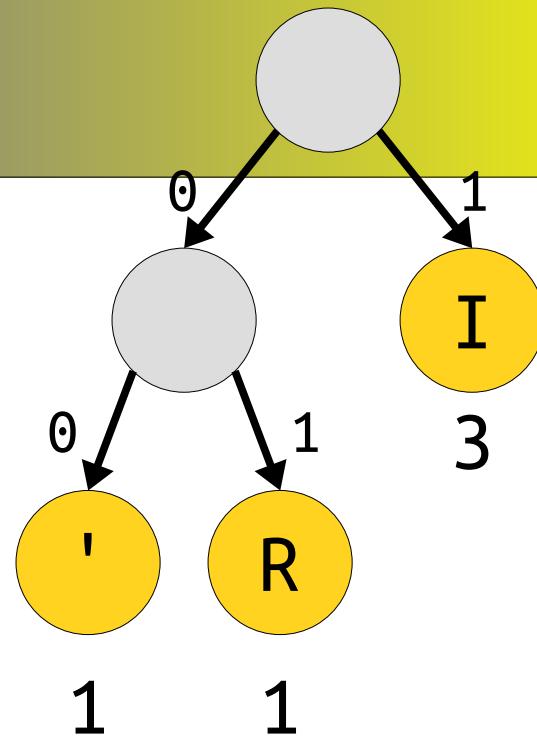
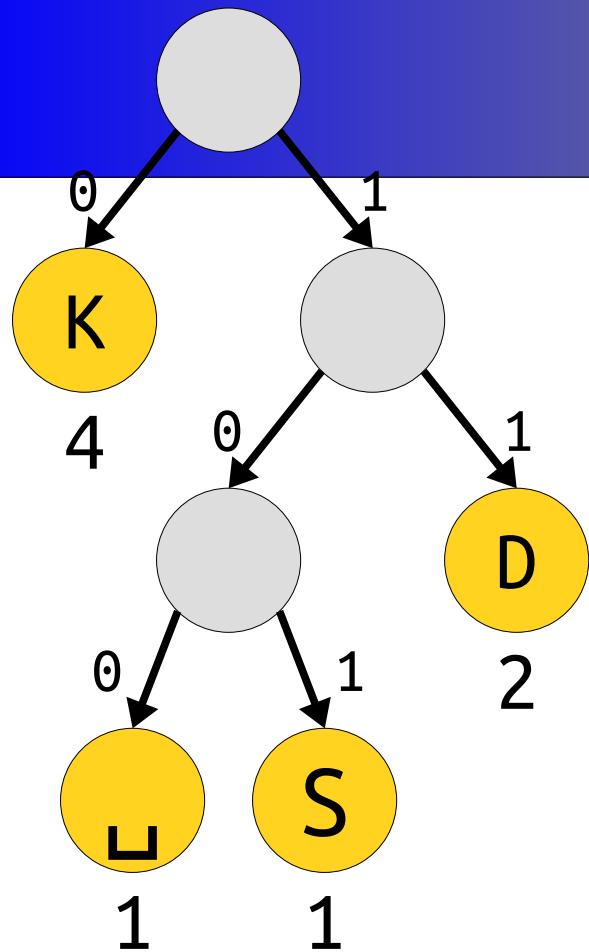


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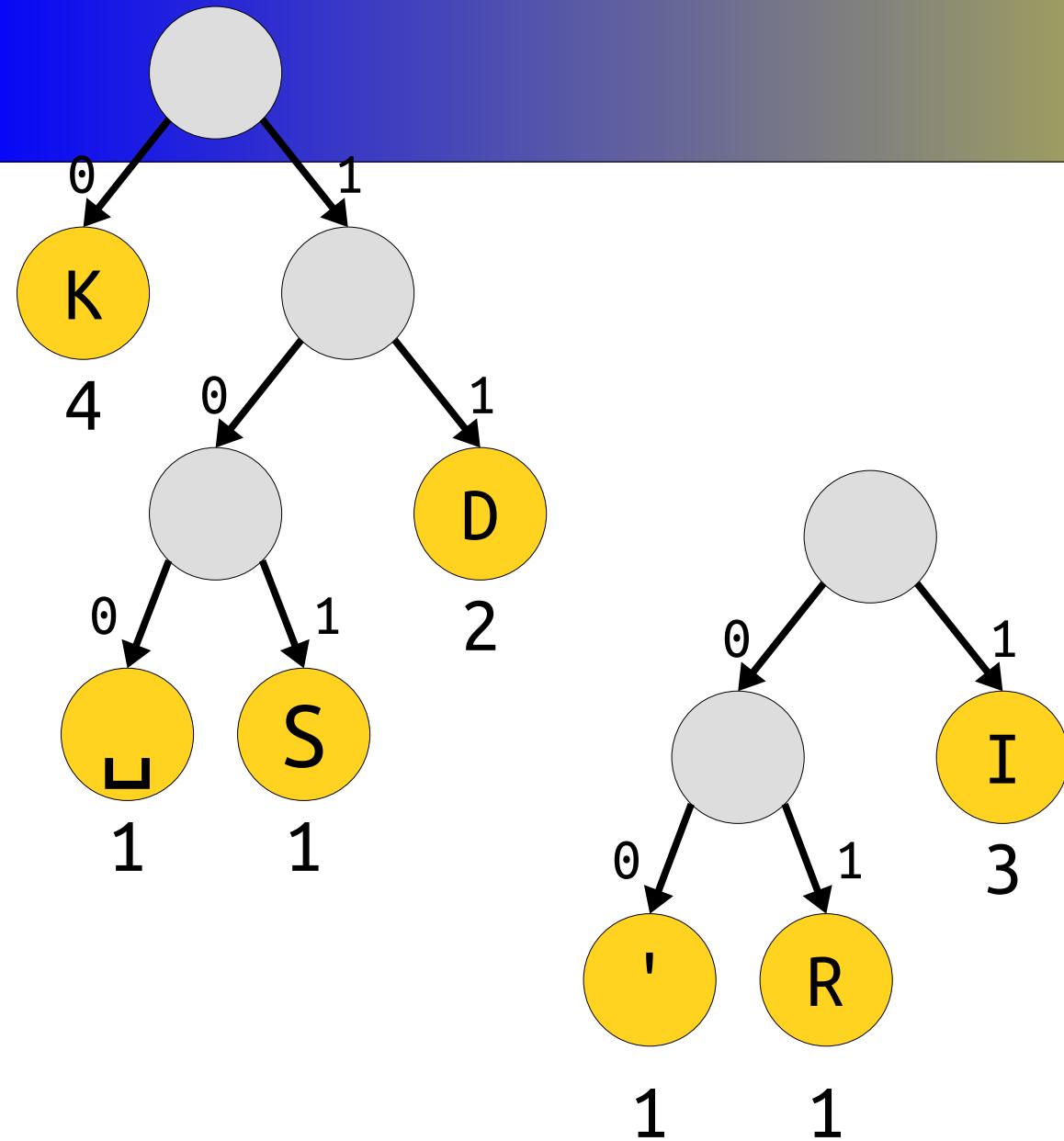


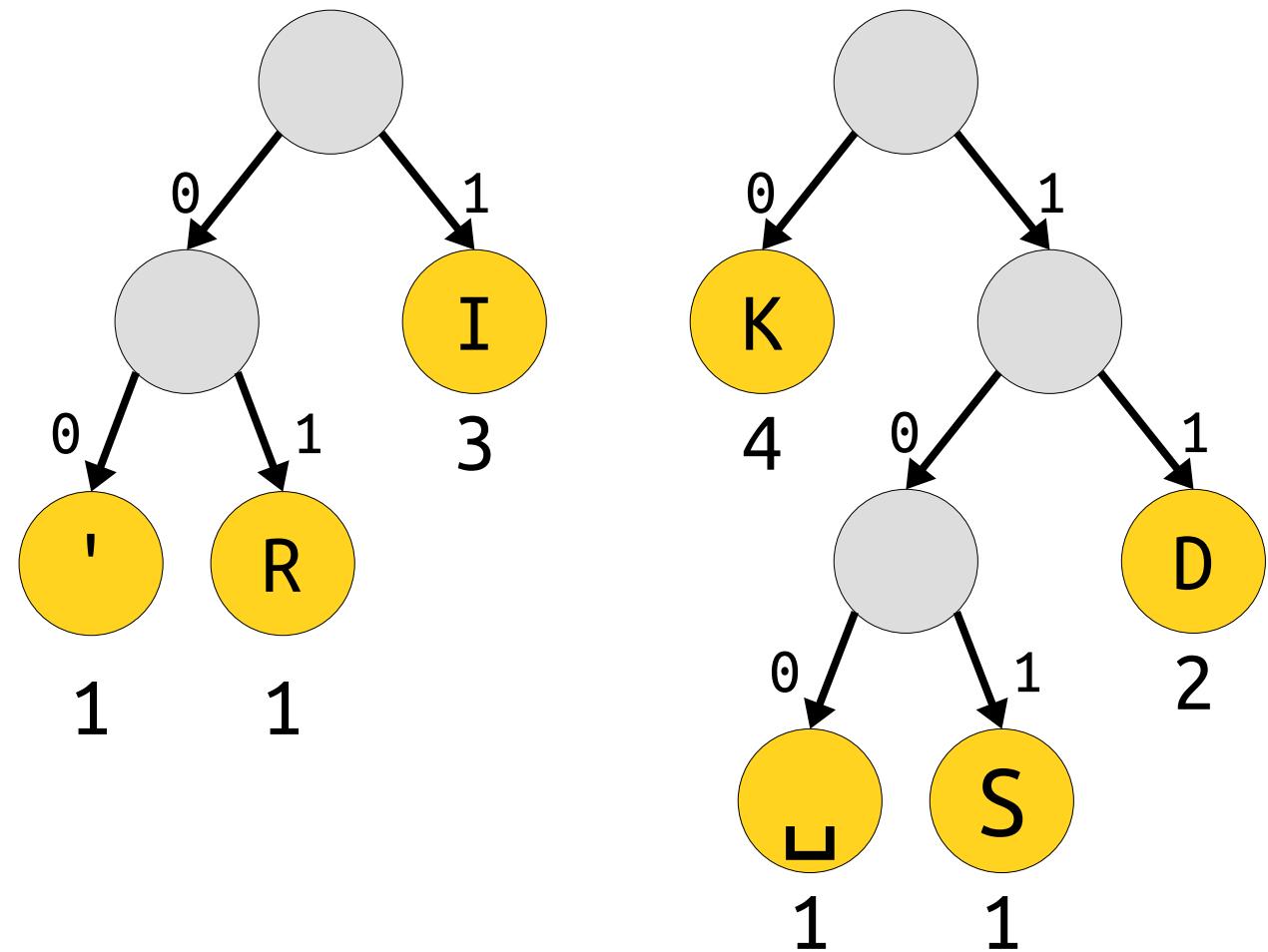
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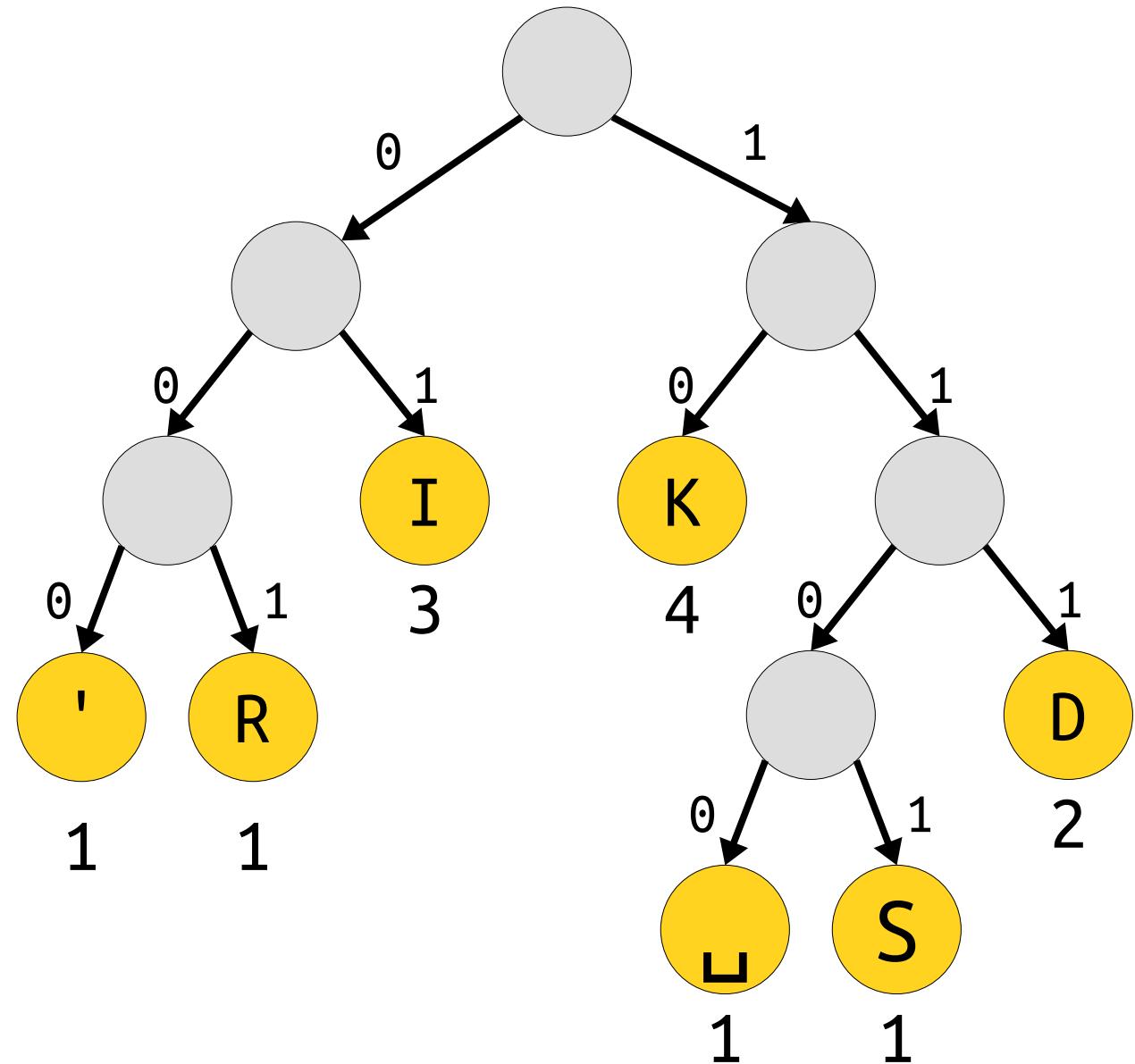
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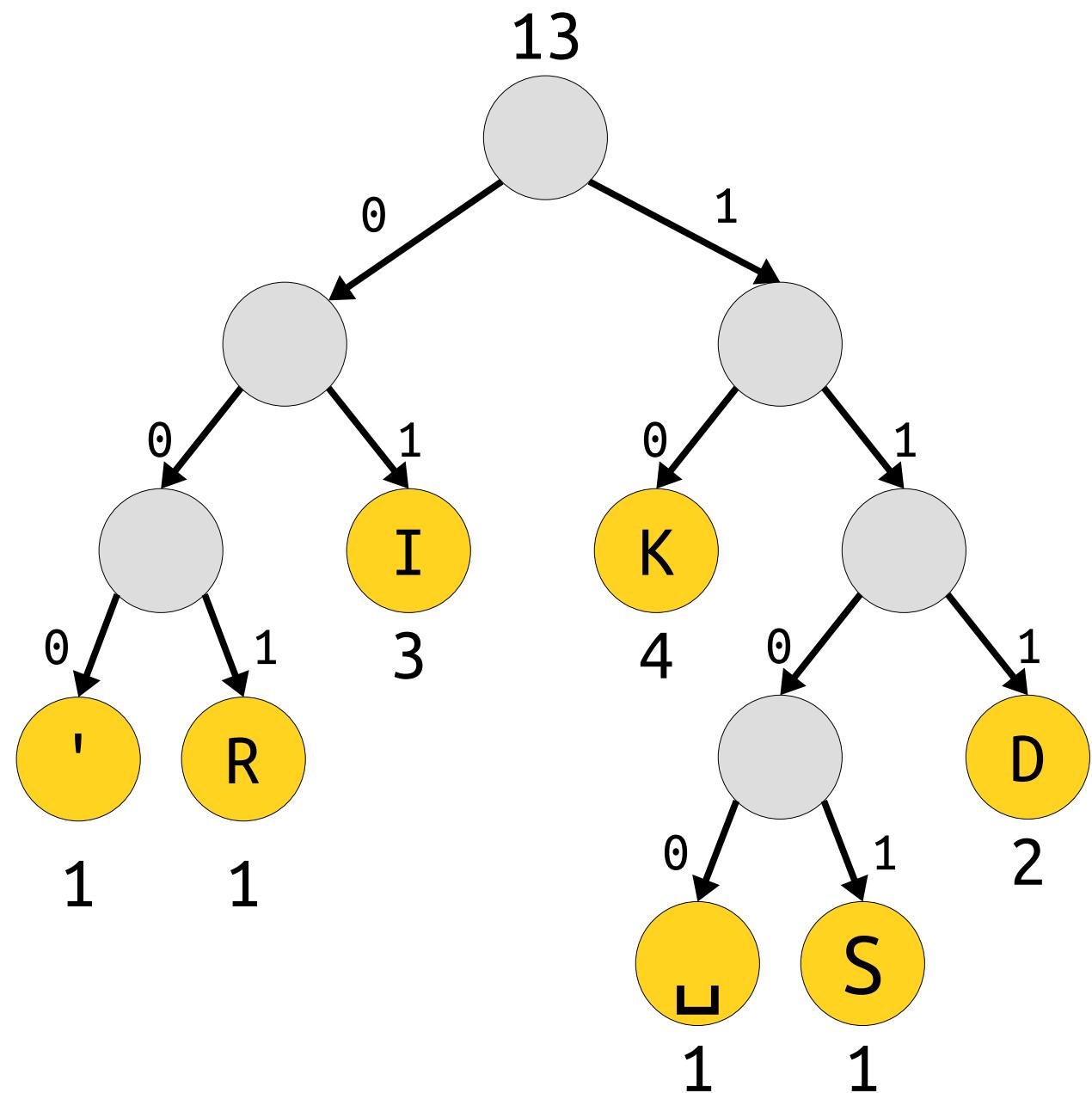


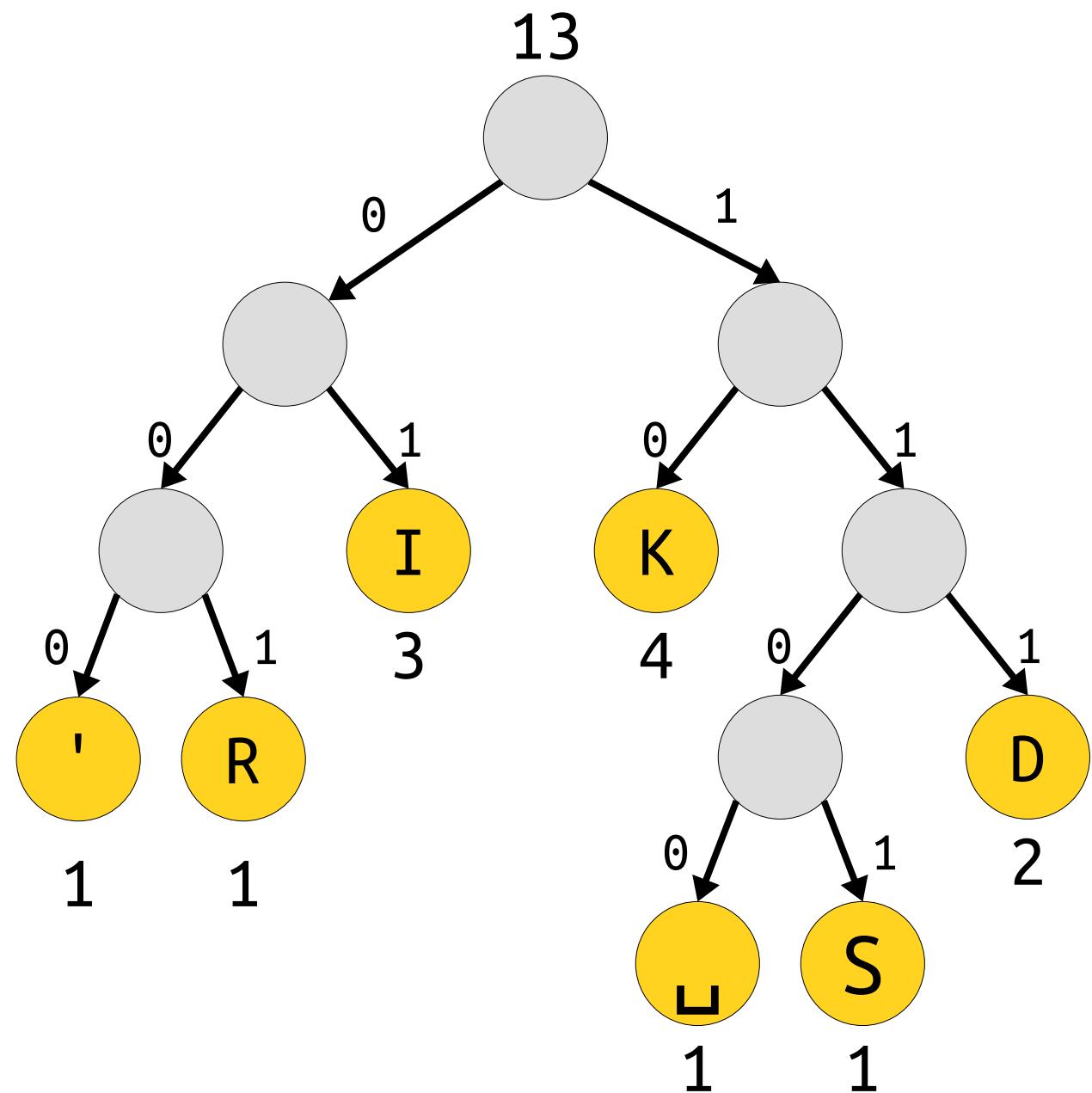
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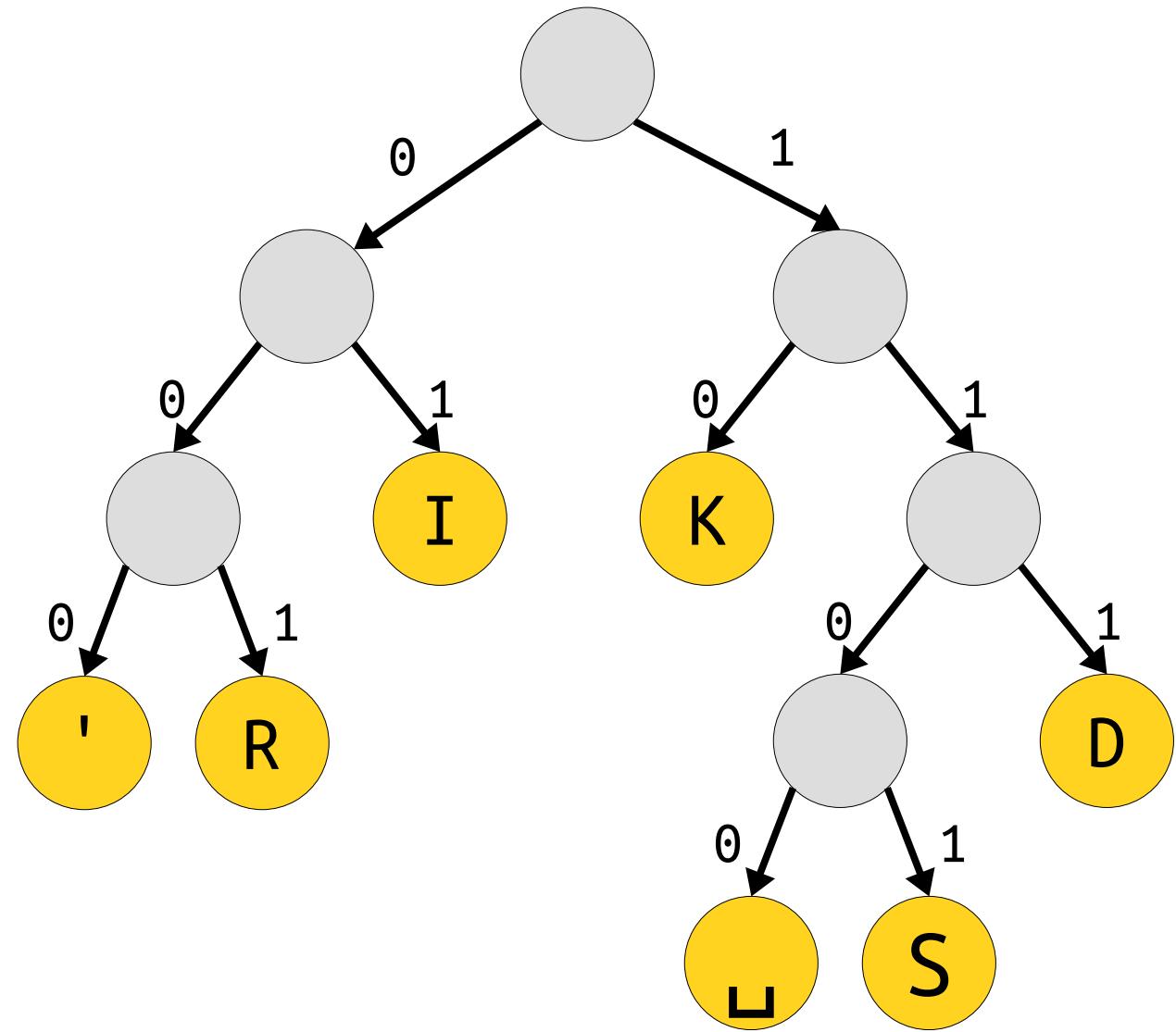






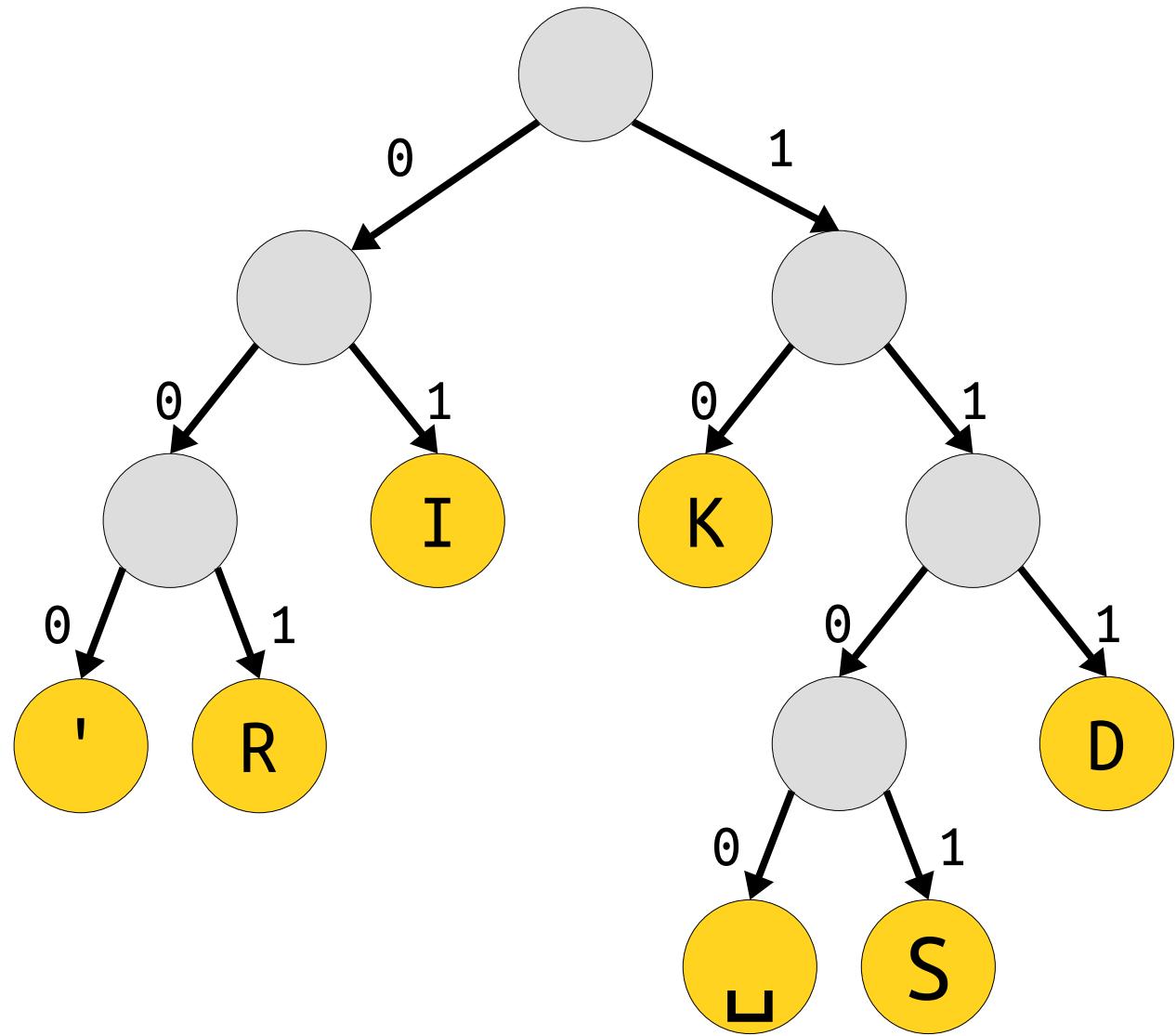






*character*      *code*

<b>K</b>	<b>10</b>
<b>I</b>	<b>01</b>
<b>D</b>	<b>111</b>
<b>R</b>	<b>001</b>
<b>'</b>	<b>000</b>
<b>S</b>	<b>1101</b>
<b>L</b>	<b>1100</b>



# ★ Huffman Coding ★

- Create a priority queue that holds partial trees.
- Create one leaf node per distinct character in the input string. The weight of that leaf is the frequency of the character. Add each to the priority queue.
- While there are two or more trees in the priority queue:
  - Dequeue the two lowest-priority trees.
  - Combine them together to form a new tree whose weight is the sum of the weights of the two trees.
  - Add that tree back to the priority queue.

# An Important Detail

# Prefix-Free Codes

*character      code*

<b>K</b>	<b>10</b>
<b>I</b>	<b>01</b>
<b>D</b>	<b>111</b>
<b>R</b>	<b>001</b>
<b>'</b>	<b>000</b>
<b>S</b>	<b>1101</b>
<b>L</b>	<b>1100</b>

# Prefix-Free Codes

*character      code*

<b>K</b>	<b>10</b>
<b>I</b>	<b>01</b>
<b>D</b>	<b>111</b>
<b>R</b>	<b>001</b>
<b>'</b>	<b>000</b>
<b>S</b>	<b>1101</b>
<b>„</b>	<b>1100</b>

<b>10</b>	<b>01</b>	<b>001</b>	<b>10</b>	<b>000</b>	<b>1101</b>	<b>1100</b>	<b>111</b>	<b>01</b>	<b>10</b>	<b>111</b>	<b>01</b>	<b>10</b>
<b>K</b>	<b>I</b>	<b>R</b>	<b>K</b>	<b>'</b>	<b>S</b>	<b>„</b>	<b>D</b>	<b>I</b>	<b>K</b>	<b>D</b>	<b>I</b>	<b>K</b>

# Prefix-Free Codes

*character*      *code*

<b>K</b>	<b>10</b>
<b>I</b>	<b>01</b>
<b>D</b>	<b>111</b>
<b>R</b>	<b>001</b>
<b>'</b>	<b>000</b>
<b>S</b>	<b>1101</b>
<b>„</b>	<b>1100</b>

1001001100001101110011101101110110

<b>10</b>	<b>01</b>	<b>001</b>	<b>10</b>	<b>000</b>	<b>1101</b>	<b>1100</b>	<b>111</b>	<b>01</b>	<b>10</b>	<b>111</b>	<b>01</b>	<b>10</b>
<b>K</b>	<b>I</b>	<b>R</b>	<b>K</b>	<b>'</b>	<b>S</b>	<b>„</b>	<b>D</b>	<b>I</b>	<b>K</b>	<b>D</b>	<b>I</b>	<b>K</b>

# Prefix-Free Codes

*character*      *code*

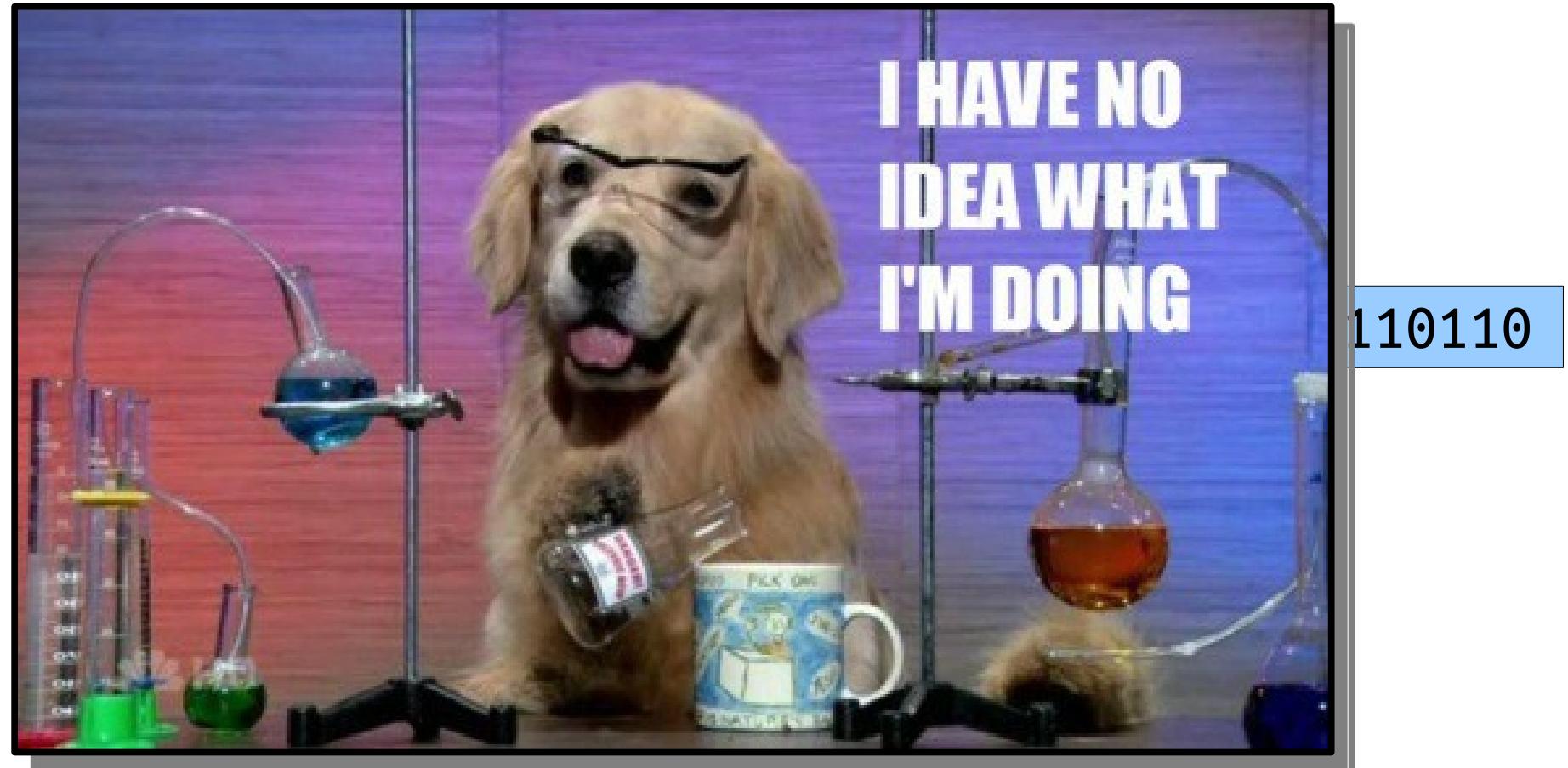
<b>K</b>	<b>10</b>
<b>I</b>	<b>01</b>
<b>D</b>	<b>111</b>
<b>R</b>	<b>001</b>
<b>'</b>	<b>000</b>
<b>S</b>	<b>1101</b>
<b>L</b>	<b>1100</b>

1001001100001101110011101101110110

# Prefix-Free Codes

```
1001001100001101110011101101110110
```

# Prefix-Free Codes



# Transmitting the Tree

- In order to decompress the text, we have to remember what encoding we used!
- **Idea:** Prefix the compressed data with a header containing information to rebuild the tree.

*Encoding Tree*

11011100101110111100010011010101110...

- This might increase the total file size!
- **Theorem:** There is no compression algorithm that can always compress all inputs.
  - **Proof:** Take CS103!

# Summary of Huffman Coding

- Prefix-free codes can be modeled as binary trees with characters in the leaves.
- Huffman coding assembles an encoding tree by repeatedly combining the lowest-frequency trees together until only one tree remains.
- We need to send the encoding table with the compressed message for it to be decoded, which can increase file sizes.

# More to Explore

- ***Kolmogorov Complexity***
  - What's the theoretical limit to compression techniques?
- ***Adaptive Coding Techniques***
  - Can you change your encoding system as you go?
- ***Shannon Entropy***
  - A mathematical bound on Huffman coding.
- ***Binary Tries***
  - Other applications of trees like these!

# Your Action Items

- ***Read the Guide to Huffman Coding***
  - It's a useful companion to this lecture.
- ***Start Assignment 8.***
  - It's a fun one! We think you'll really like it.
  - Aim to complete Milestone One by Monday.

# Next Time

- ***Graphs***
  - Representing networks of all sorts.
- ***Graph Searches***
  - A new perspective on some earlier ideas.

# Appendix: ***UTF-8***

# Beyond ASCII

- ASCII was invented in 1960s America, when the main concern was storing English text.
- It's completely inadequate for storing the rich breadth of characters that actually get used across the whole world in the 2020s.
- What are we using now?

# Unicode

- **Unicode** is a system for representing glyphs and symbols from all languages and disciplines.
- One of the most common encodings is **UTF-8**, which uses sequences of bytes to represent any one individual character.
- The basic idea:
  - UTF-8 is a prefix code, so less common characters like and € use more bits than common characters like e and 你.
  - UTF-8 encodings are always a full multiple of 8 bits long, making it easier for computers to work one byte at a time.
  - UTF-8 is backwards-compatible with ASCII, so any text encoded with ASCII is also valid UTF-8.

# UTF-8

Option 1

0ddddddd

Option 2

110ddddd 10dddddd

Option 3

1110dddd 10dddddd 10dddddd

Option 4

11110ddd 10dddddd 10dddddd 10dddddd

# UTF-8

```
1111000010011111001010110001100
```

# UTF-8

```
11110000 10011111 10010101 10001100
```

# UTF-8

11110000	10011111	10010101	10001100
11110000	10011111	10010101	10001100

# UTF-8

11110000	10011111	10010101	10001100
<u>11110000</u>	<u>10011111</u>	<u>10010101</u>	<u>10001100</u>

# UTF-8

11110000 10011111 10010101 10001100

11110000 10011111 10010101 10001100

000001111010101001100

# UTF-8

11110000 10011111 10010101 10001100

11110000 10011111 10010101 10001100

000001111010101001100